Linear Algebra Notes

The **matrix multiplication** A B requires that the number of columns in A must be the same as the number of rows in B, i.e. if A is m-by-p the B must be p-by-n. Alternately we could say that the length of the rows of A must be the same as the length of the columns of B. The result of multiplying an m-by-p matrix A times a p-by-n matrix B is an m-by-n matrix C in which the (i,j) element is the sum of the products of the elements of the ith row of A and the jth column of B. For example

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 5 \\ 6 \end{bmatrix} \text{ yields the 2-by-1 product } \begin{bmatrix} 17 \\ 39 \end{bmatrix}$$

Matrix multiplication is not commutative, i.e. generally A*B $\neq$B*A.

The common method of **solving a system of linear equations** is to write the augmented matrix for the system and then use elementary row operations to reach an equivalent system of equations in **row reduce echelon form**. This method is called the Gauss-Jordan elimination method. The MATLAB function that does the reduction to reduced row echelon form is rref(). The solutions to the system of equations can be read from the reduced row echelon form. For example, the system of equations:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 17 \\ 39 \end{bmatrix}$$

has the augmented matrix $\begin{bmatrix} 1 & 2 & 17 \\ 3 & 4 & 39 \end{bmatrix}$ which has the reduced row echelon form

$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \end{bmatrix}$ which corresponds to $x_1 = 5$ and $x_2 = 6$.

The **inverse of a square matrix** A is the unique matrix B for which AB = I = BA where I is the identity matrix. If a square matrix A has an inverse we call it invertible or **nonsingular** otherwise it is **singular** or noninvertible. The inverse matrix can also be used to solve systems of equations. Specifically, if AX = C and A is an n-by-n invertible matrix then X = BC where B is the inverse of A For example, if A is the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then it inverse matrix is $\begin{bmatrix} -2 & 1 \\ 1.5 & -.5 \end{bmatrix}$ and the solution to the system of equations above is computed as $\begin{bmatrix} -2 & 1 \\ 1.5 & -.5 \end{bmatrix} \begin{bmatrix} 17 \\ 39 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$.

If square matrix A has a scalar λ and a vector X for which

$$AX = \lambda X$$

Then we say λ is an **eigenvalue** for A with the associated **eigenvector** X. In MATLAB eig(A) returns a column vector of the eigenvalues of A.

For example: if P is the matrix

$$\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}$$

and X is the vector $\begin{bmatrix} 7000 \\ 10000 \\ 4000 \end{bmatrix}$

then P*X is $\begin{bmatrix} 7000 \\ 10000 \\ 4000 \end{bmatrix} = X$

which means that 1 is an eigenvalue of P with corresponding eigenvector $\begin{bmatrix} 7000 \\ 10000 \\ 4000 \end{bmatrix}$.