M260 2.1

P. Staley

Predicates and Quantified Statements I

A __________________ is a sentence that contains a finite number of ________________ and becomes a statement when specific values are substituted for the _________________.

The domain of a __________________ _______________ is the set of all values that may be substituted in place of the __________________.

Consider the predicate P, "--- is a student at ---". P(x,y) becomes "x is a student at y".

The _______________ of the first variable, x, is the set of ________________ and the _______________ of the second variable y is the set of __________________.

P(x,y) is not a statement because it ____.

P(Iliana, Southwestern College) is the statement ______________________ _______ _____________________________ ______________. P(Iliana, Stanford) is the statement ______________________________ ____________________________.

If Q(x) is a _____________________ and x has ____________________ D, then the truth set of _______ is the set of elements of _____ that make Q(x) ________ when substituted for x.

The truth set of Q(x) is denoted

$$\{x \in D \mid Q(x)\}$$

which is read: "____."

Let P(x) and Q(x) be ____________________ and suppose the common domain of x is D.

The notation $P(x) \Rightarrow Q(x)$ means that ____ ______________________________________ ____. The notation $P(x) \Leftrightarrow Q(x)$ means that ___________________________________ __________________________________.

To change a predicate into a statement we can assign values to the __________________ .

Another way to obtain statements from predicates is to add ________________.

The symbol ∀ denotes "________________" and is called the _______________ _______________. Examples: "∀ human beings x, x is mortal" or "∀ x ∈ (the set of human beings), x is mortal".

Let Q(x) be a predicate and D the ___________ of x. A universal statement is a statement of the form "_________________________________." It is defined to be true _______________________ _______________ _______________. It is defined to be false ______________ _______ _____________________________. A value of x for which Q(x) is false is called a _________________________. The method of _________________________ consists of showing the truth of the predicate separately for each individual element of the domain.

The symbol ∃ denotes "________________" and is called the _______________ _______________. Example: "∃ a person s such that s is passing Math 260 this semester". Let Q(x) be a predicate and D the ___________ of x. An existential statement is a statement of the form "_________________________________." It is defined to be true __________________________________ _______________________ _________.

It is defined to be false _________ ______ ______________________________________.

The universal conditional statement has the form

____.

Write the following as formal universal conditional statements:

a. If a real number has a terminating decimal representation then it is a rational number.

____.

b. All students are hard workers.

____.

c. There are no bad teachers at Southwestern College.

____.

The negation of a statement of the form: $\forall x$ in D, Q(x)

is logically equivalent to a statement of the form

______________________________________.

Symbolically: $\sim(\forall x \in D, Q(x)) \equiv$ ___________________________.

The negation of a universal statement ("all are") is logically equivalent to an existential statement ("some are not").

The negation of a statement of the form: $\exists x$ in D such that Q(x)

is logically equivalent to a statement of the form

______________________________________.

Symbolically: $\sim(\exists x \in D$ such that $Q(x)) \equiv$ ___________________________.

The negation of an existential statement ("some are") is logically equivalent to a universal statement ("all are not").

Give the formal negation of the statement: ∀even integers n, n^2 ? n.

____

Give the formal negation of the statement: ∃ a student s such that s does not have an email account.

____

The negation of the universal quantifier is given as

~(∀x, if P(x) then Q(x)) ≡ ____

or symbolically

~(∀x, P(x) → Q(x)) ≡ ____

Write a formal negation for: ∀student s, if s filled out the notes sheet for exam one then s got at least a C on exam one. ____

____

____ .