M250 Practice Exam 4.4-5.2
P. Staley

Definitions, integration by substitution, ln x, and numerical integration. Show your work on 7-15.

1. State the definition for the derivative of f(x):

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. State the definition for the definite integral of f(x) on the interval [a,b]:

The limit of the Riemann Sums as the norm of the partition goes to zero.

3. State the definition for the indefinite integral of f(x):

The most general antiderivative.

4. State the Fundamental Theorem of Calculus:

For $f(x)$ continuous on $[a, b]$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

5. What is the average of f(x) on the interval [a,b]?

$$\frac{1}{b-a} \int_a^b f(x)\,dx$$

6. What is the definition for ln(x) as given in the calculus class:

$$\ln(x) \equiv \int_1^x \frac{1}{t}\,dt$$

7. Use the Trapezoid rule and Simpson's rule with n=6 to evaluate the integral:

$$\int_{0.4}^{1.0} (1/t)dt$$

$\Delta x = \frac{b-a}{n} = \frac{1.0-0.4}{6} = 0.1$

i	0	1	2	3	4	5	6
x_i	.4	.5	.6	.7	.8	.9	1.0
$f x_i$	$1/.4$	$1/.5$	$1/.6$	$1/.7$	$1/.8$	$1/.9$	$1/1.0$

Trap $\frac{1.0-0.4}{2\cdot 6}\left(\frac{1}{.4} + 2\cdot\frac{1}{.5} + 2\cdot\frac{1}{.6} + 2\cdot\frac{1}{.7} + 2\cdot\frac{1}{.8} + 2\cdot\frac{1}{.9} + \frac{1}{1.0}\right) = .920635$

Simpson $\frac{1.0-0.4}{3\cdot 6}\left(\frac{1}{.4} + 4\cdot\frac{1}{.5} + 2\cdot\frac{1}{.6} + 4\cdot\frac{1}{.7} + 2\cdot\frac{1}{.8} + 4\cdot\frac{1}{.9} + \frac{1}{1.0}\right) = .916402$

exact value: $\ln(1) - \ln .4 = \ln 2.5 \approx .916291$

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8. $\int \frac{x\sin(x^2)}{\cos(x^2)}dx$

$u = \cos(x^2)$

$du = -2x\sin(x^2)dx$

$\frac{du}{-2} = x\sin(x^2)dx$

$-\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|\cos(x^2)| + c$

or $\frac{1}{2}\ln|\sec(x^2)| + c$

or $\ln\sqrt{|\sec(x^2)|} + c$

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9. $\int x\sqrt{x+5}dx$

$u = x+5$

$du = dx$

$x = u-5$

$\int (u-5)\sqrt{u}\,du = \int u^{1.5} - 5u^{.5}\,du$

$= \frac{u^{2.5}}{2.5} - \frac{5u^{1.5}}{1.5} = \frac{2(x+5)^{5/2}}{5} - \frac{10(x+5)^{3/2}}{3} + c$

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10. $\int \frac{\sec(x)\tan(x)}{\sec(x)}dx = \int \frac{du}{u} = \ln|u| + c$

$= \ln|\sec(x)| + c$

$u = \sec(x)$

$du = \sec(x)\tan(x)dx$

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11. $\int (\sin^2 x + 1/\sin x)\cos(x)dx = \int \left(u^2 + \frac{1}{u}\right)du = \frac{u^3}{3} + \ln|u| + c$

$u = \sin(x)$

$du = \cos(x)dx$

$= \frac{\sin^3(x)}{3} + \ln|\sin(x)| + c$

12. $\int_3^{27} (1/t)dt = \left[\ln t\right]_3^{27}$

$= \ln 27 - \ln 3$

$= \ln 9$

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13. The average of y = 1/x on [1,e] is

$$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx = \frac{1}{e-1}\left(\ln e - \ln 1\right) = \frac{1}{e-1}$$

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14. $\int x\sec^2(x^2+2)dx = \frac{1}{2}\int \sec^2(u)\,du = \frac{1}{2}\tan u + c$

$u = x^2+2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\tan(x^2+2) + c$

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15. $\frac{d}{dx}\ln|\cos(x)| = \frac{1}{\cos(x)}(\cos(x))' = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$