How Letters Are Used in Mathematics

Understanding the customary usage of letters in mathematics can greatly assist in interpreting the import of an equation or expression. For example—if there is a simple mixing of a,b,c, and x it is most likely that x represents a variable of unknown or unspecified value where as the a, b, and c are likely to be place holders for values that do not change. This article discusses the customary uses of letters in lower division mathematics.

Constants. π is used for the ratio of the circumference of a circle to its diameter. This value is constant for all circles (3.14159…). e is used for Euler's number (2.7182818…). i is used for the imaginary number $\sqrt{-1}$. Note that this is different from the custom in engineering where j is used for $\sqrt{-1}$. $\aleph_0$ (aleph-naught) is the first transfinite number, i.e. denumerably infinite. $\mathfrak{c}$ is used for the cardinality of the continuum, i.e. $2^{\aleph_0}$.

Unspecified Constants. The letters at the beginning of the alphabet, in particular a, b, and c, are used as placeholders for Real valued constants. If the emphasis is on the form of the equation then A, B, and C are often used. For integer valued constants we generally use n then m. N is often used to indicate an arbitrarily large integer. In a number theory setting p and q are used for primes; q and r are used for quotient and remainder.

Real Variables. Real valued quantities which are unknown and expected to vary are represented by letters at the end of the alphabet. Unknowns with no specific environment are assigned in the order x, y, z, w, u, and v. The understanding is as follows:
Only x in an equation—x represents the values which make the equation true.
Only x in an expression—function definition with x as the independent variable and domain as all values for which the expression evaluates to a Real number.
Only x and y in an equation—(1) y is a (perhaps implicitly specified) function of x, or (2) (x,y) are the ordered pairs that make the equation true.
Only x and y in an expression—a function of two variables
Only x, y, and z in an equation—z is a function of the two independent variables x and y.
x, y, z, and w in an equation—w is a function of the three independent variables x, y, and z.
x,y and u,v—this is used for a mapping from $R^2 \rightarrow R^2$. (x,y) is in the domain (independent variables) and (u,v) is the range (dependent variables).

Real Variables with Understandings. Unknown values from some special domains often have customary letters: t for time, s for arc length or linear position, r for radius (use ρ if r is already taken), A for area, V for volume, e for eccentricity, θ for an angle (particularly the counterclockwise angle from the positive x-axis in the xy plane), other angle variables include ϕ and ψ. For very small unknowns use ε then δ. λ is used for unknown eigenvalues or for a wave length. ω is used for frequency as in $\sin(\omega t)$. For curvature use κ. For the angle measured down from the positive z-axis use ϕ. For the standard deviation of a population use σ. For the mean of a population use μ. For the

standard deviation of a sample use s. Δ is used as a prefix to indicate "a small change in". For the sides and opposite angles of a triangle it is common to use A, B, C with a, b, c (or a, b, c, with α, β, γ). Direction cosine angles in three space with respect to x, y, z are given by α, β, and γ respectively.

Integer Variables. Integer variables are generally assigned in the order i, j, and k.

Complex Variables. z is used instead of x as the primary unknown for complex variables. a and b are used as the real and imaginary parts of a complex number. θ is used as the principle angle. i is used as the imaginary constant.

Functions. Real valued functions are assigned in the order f, g, then h. When the emphasis is less on the mapping aspect of the function then common usage is-- y as a function of x; u as an intermediate function of x; z as a function of x and y; and w as a function of x, y, and z. For "parametric equations" x, y, and z become the dependent variables of the independent variables s, t, s and t, or u and v. In this case s is preferred for arc length and t is preferred for time. P(event description) is the probability of that event. N(,) is the Normal (Gaussian) distribution with parameters mean and standard deviation. B(,) is the Binomial distribution with parameters p (success probability) and n (number of trials). Polynomial functions often are given with p(x) and q(x) or P(x) and Q(x). δ is used for the dirac delta operator ($\delta(i,j) = 1$ if i=j, zero otherwise).

Operators. The term "operator" is used for functions whose domain is other functions. d/dx is derivative with respect to x, d/dt is derivative with respect to t, etc. d is the differential operator. Suffix ' is used when the "with respect to" variable is understood as in y' means the derivative of y with respect to x. Σ is the summation operator. Π is the product operator. For general matrices use A, B, C in that order. For linear transformations use T or L.

Special Usages. o (letter oh) and l (letter el) are avoided because of their potential confusion with 0 (number zero) and 1 (number one). If letter l is required then it is often given in a script font as ℓ. Letter like symbols that resemble R, C, Q, or Z (e.g. $\Re$) signify the Real numbers, Complex numbers, Rational numbers, or Integers respectively. ϕ or Φ is used for the empty set.