Review from First Semester Calculus

M251 P. Staley

1-6 State the definition for each of the following

1. The function f(x) is continuous at c $\lim_{x \to c} f(x) = f(c)$

3. The derivative of f(x) with respect to x in terms of a limit: $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

4. The definite integral of f(x) on [a,b] in terms of a limit:

The limit of the Riemann Sums as the norm of the partition goes to zero.

5. The indefinite integral of f(x):

The most general antiderivative

6. ln(x) in terms of an integral:

$\ln(x) = \int_1^x \frac{1}{t} dt \quad \text{for } x > 0$

7. The average of f(x) on the interval [a,b]:

$\frac{1}{b-a} \int_a^b f(x)\, dx$

8. State the Fundamental Theorem of Calculus:

For f(x) continuous on [a,b]

$\int_a^b f(x)\, dx = F(b) - F(a)$

where F(x) is any antiderivative of f(x).

9. Omit

10. Omit

11. $\int x^n dx =$ $\frac{x^{n+1}}{n+1} + c$ for $n \neq -1$

12. $\int \sin(x) dx =$ $-\cos x + c$

13. $\int \cos(x) dx =$ $\sin x + c$

14. $\int \frac{1}{x} dx =$ $\ln|x| + c$

15. $\int e^x dx =$ $e^x + c$

16. $\frac{d}{dx} x^n =$ $n x^{n-1}$

17. $\frac{d}{dx} \sin(x) =$ $\cos x$

18. $\frac{d}{dx} \cos(x) =$ $-\sin x$

19. $\frac{d}{dx} \tan(x) =$ $\sec^2 x$

20. $\frac{d}{dx} \sec(x) =$ $\sec x \tan x$

21. $\frac{d}{dx} \ln(x) =$ $\frac{1}{x}$

22. $\frac{d}{dx} e^x =$ e^x

23. $d(y^n) =$ $n y^{n-1} dy$

24. $d(\sin(\theta)) =$ $\cos\theta \, d\theta$

25. $d(\cos(z)) =$ $-\sin z \, dz$

26. $d(\tan(\psi)) =$ $\sec^2\psi \, d\psi$

27. $d(\sec(x)) =$ $\sec x \tan x \, dx$

28. $d(\ln(u)) =$ $\frac{1}{u} du$

29. $d(e^x) =$ $e^x dx$

30. $d(3\pi^2) =$ 0

31-35 Integrate the following:

31. $\int \tan^3(x)\sec^2(x)dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^3 du = \frac{u^4}{4} + C = \frac{\tan^4 x}{4} + C$

32. $\int (\sin^3(x)+1)\cos(x)dx$

$u = \sin(x)$

$du = \cos(x)dx$

$\int u^3 + 1\,du$

$\frac{u^4}{4} + u + C = \frac{\sin^4 x}{4} + \sin x + C$

33. $\int \frac{t^2+2t+1}{\sqrt{t+2}}dt$

$u = t+2$

$du = dt$

$\int \frac{(u-2)^2 + 2(u-2)+1}{\sqrt{u}}du = \int \frac{u^2-4u+4+2u-4+1}{\sqrt{u}}du$

$= \int u^{1.5} - 2u^{.5} + u^{-.5}\,du = \frac{u^{2.5}}{2.5} - \frac{2u^{1.5}}{1.5} + \frac{u^{.5}}{.5} + C$

$\left(\frac{2(t+2)^2}{5} - \frac{4(t+2)}{3} + \frac{1}{.5}\right)\sqrt{t+2} + C$

$\left(\frac{2}{5}t^2 - \frac{8}{15}t + \frac{2}{15}\right)\sqrt{t+2} + C$

$\left(3t^2 - 4t + 1\right)\frac{2}{15}\sqrt{t+2} + C$

34. $\int e^{\sin(x)}\cos(x)dx$

$u = \sin(x)$

$du = \cos x\,dx$

$\int e^u du = e^u = e^{\sin(x)} + C$

35. $\int_1^2 \frac{2x+1}{x^2+x}dx$

$u = x^2 + x$

$du = (2x+1)dx$

$\int_2^6 \frac{du}{u} = [\ln u]_2^6 = \ln 6 - \ln 2 = \ln 3$