M250 Final Exam Practice
Answers

$\int \frac{\ln x}{x}\,dx$ $\quad u=\ln x \quad du=\frac{dx}{x}$ $\quad \int u\,du = \frac{u^2}{2}+c = \frac{(\ln x)^2}{2}+c$

$\int \frac{1}{x(\ln x)^2}\,dx$ $\quad u=\ln x \quad du=\frac{dx}{x}$ $\quad \int \frac{1}{u^2}\,du = \frac{u^{-1}}{-1}+c = \frac{-1}{\ln x}+c$

$\int \frac{\cos x}{\sin^3 x}\,dx$ $\quad u=\sin x \quad du=\cos x\,dx$ $\quad \int u^{-3}\,du = \frac{-1}{2u^2}+c = \frac{-\csc^2(x)}{2}+c$

$\int \frac{1}{x\ln(x^2)}\,dx$ $\quad u=\ln x \quad du=\frac{dx}{x}$ $\quad \int \frac{1}{2u}\,du = \frac{1}{2}\ln|u|+c$

$= \frac{1}{2}\ln|\ln(x)|+c$

$\int x e^{-x^2}\,dx$ $\quad u=-x^2 \quad du=-2x\,dx$

$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^{-x^2}+c$

$$u = -(x^3+3x)$$
$$du = -3(x^2+1)\,dx$$

$$\int \frac{x^2+1}{e^{x^3+3x}}\,dx \qquad -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^{-(x^3+3x)} + C$$

$$u = e^\theta$$
$$du = e^\theta d\theta$$

$$\int e^\theta \sec^2(e^\theta)\,d\theta$$

$$\int \sec^2 u\,du = \tan(u) + C$$
$$= \tan e^x + C$$

$$\frac{d}{dz} e^{\ln z} = \frac{dz}{dz} = 1$$

$$\frac{d}{dz} e^{z^2-\ln z} = (z^2-\ln z)' e^{z^2-\ln z}$$
$$= \left(2z - \frac{1}{z}\right) e^{z^2-\ln z}$$

$$\frac{d}{dx} e^{\sec\theta} = \sec\theta \tan\theta \, e^{\sec\theta}$$

$$\frac{d}{dx}\left(x^3+3^x+3^3\right) = 3x^2+ (\ln 3)\, 3^x$$

$$\int x^3+3^x+3^3 \, dx = \frac{x^4}{4} + \frac{3^x}{\ln 3} + 27x + C$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\cos^2\theta} \, d\theta = 2\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$$

$$= 2\left[-\cos\theta\right]_0^{\frac{\pi}{2}} = 2(0--1) = 2$$

$u = z^2$
$du = 2z\,dz$

$$\int_0^3 z e^{z^2}\,dz = \frac{1}{2}\int_0^9 e^u\,du = 2\left[e^u\right]_0^9 = 2\left(e^9 + e^0\right)$$
$$= 2\left(e^9 - 1\right)$$

$$\int_0^{\frac{\pi}{4}} (\ln\cos\theta)\tan\theta\,d\theta$$

$u = \ln\cos\theta$
$du = -\tan\theta\,d\theta$

$$-\int_0^{\ln\left(\frac{1}{\sqrt{2}}\right)} u\,du$$

$$= -\left[u^2\right]_0^{\ln\frac{1}{\sqrt{2}}} = -\left(\ln\frac{1}{\sqrt{2}}\right)^2$$
$$= -\left(-\frac{1}{2}\ln 2\right)^2$$
$$= -\frac{(\ln 2)^2}{4}$$

$$\int_{-.37}^{.37} x \sin x^2\,dx$$

odd odd even $\Rightarrow$ odd

hence $\int_{-.37}^{.37} \text{odd} = 0$