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Practice Exam #1

Answers

1. $f(1)=2, f(4)=6$

$$P = (1, 2) \quad Q = (4, 6) \quad \text{Thus } M_{PQ} = \frac{6-2}{4-1} = \frac{4}{3}$$

2. When $t=1, y=64$ ft then 2 seconds later

$$t=3 \text{ and } y = 80 \cdot 3 - 16 \cdot 3^2 = 96 \text{ ft}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}} = \frac{96 - 64 \text{ ft}}{2 \text{ seconds}} = 16 \text{ ft/sec}$$

3

$$(a) \lim_{x \rightarrow 3} f(x) = 2$$

$$(b) \lim_{x \rightarrow 1} f(x) = -1$$

$$(c) \lim_{x \rightarrow -3} f(x) = 1$$

$$(d) \lim_{x \rightarrow 2^-} f(x) = 1$$

$$(e) \lim_{x \rightarrow 2^+} f(x) = 2$$

$$(f) \lim_{x \rightarrow 2} f(x)$$

limit does not exist

$$4. \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} = -1$$

$$5. \lim_{h \rightarrow 0} \frac{(h-2)^2 - 4}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 4h + 4 - 4}{h}$$

$$= \lim_{h \rightarrow 0} h - 4 = -4$$

$$6. \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$$

$$= \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{(x-4)} = \lim_{x \rightarrow 4} \sqrt{x} + 2 = 4$$

$$7. 25-.2 < 3x+4 < 25+.2$$

$$21-.2 < 3x < 21+.2$$

$$7 - \frac{2}{3} < x < 7 + \frac{2}{3}$$

so x must be within $\frac{2}{3}$ of 7

8.

$$3-\varepsilon < 2x-1 < 3+\varepsilon$$

$$4-\varepsilon < 2x < 4+\varepsilon$$

$$2 - \frac{\varepsilon}{2} < x < 2 + \frac{\varepsilon}{2}$$

$$\text{or } |x-2| < \frac{\varepsilon}{2} = \frac{1}{4} = \delta$$

9a.

$$\frac{(x+1)^2}{(x^2-1)} = \frac{(x+1)(x+1)}{(x+1)(x-1)} = \frac{x+1}{x-1} \text{ except at } x=-1$$

thus if $\frac{(x+1)^2}{x^2-1}$ is assigned the value 0 at $x=-1$

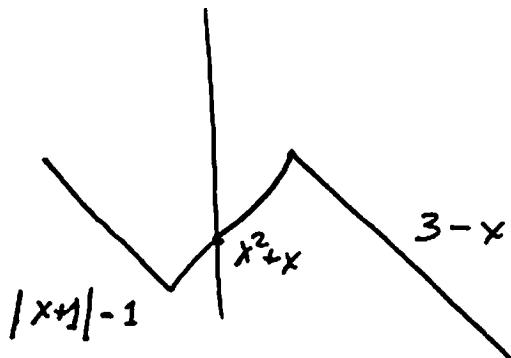
the function will be continuous at $x=-1$
ie $x=-1$ is a removable discontinuity.

9b

$$\lim_{x \rightarrow 2^+} \frac{|x-2|}{x-2} = 1 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ ie } \frac{|x-2|}{x-2} \text{ has a} \\ \lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2} = -1$$

jump discontinuity at $x=2$.

10.



continuous everywhere.

11. The instantaneous velocity v is

$$v = \lim_{h \rightarrow 0} \frac{(80(t+h) - 16(t+h)^2) - (80t - 16t^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{80t + 80h - 16t^2 - 32ht - 16h^2 - 80t + 16t^2}{h}$$

$$= \lim_{h \rightarrow 0} 80 - 32t - 16h = 80 - 32t$$

maximum height occurs for $v=0$ ie $80-32t=0$

$$\text{or } t = \frac{80}{32} = \frac{5}{2} = 2.5 \text{ seconds}$$

$$y \text{ at } t=0 \text{ is } 80 \cdot 0 - 16 \cdot 0^2 = 0$$

$$y \text{ at } t=2.5 \text{ is } 80 \cdot \frac{5}{2} - 16 \cdot \left(\frac{5}{2}\right)^2 = 100$$

Thus average velocity from $t=0$ to $t=2.5$

$$\bar{v} = \frac{100 - 0}{2.5 - 0} = 40 \text{ ft/sec}$$

12.

$$\text{slope } m = \lim_{h \rightarrow 0} \frac{5+h + \frac{1}{5+h} - \frac{26}{5}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{25+10h+h^2}{5+h} + \frac{1}{5+h} - \frac{26(5+h)}{5(5+h)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{(26+10h+h^2)5}{(5+h)5} - \frac{26(5+h)}{5(5+h)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{130+50h+5h^2}-\cancel{130}-\cancel{26h}}{5(5+h)h}$$

$$\lim_{h \rightarrow 0} \frac{24+5h}{5(5+h)} = \frac{24}{25}$$

Point slope form for the tangent line is then

$$y - \frac{26}{5} = \frac{24}{25}(x-5)$$

or simplifying

$$25y - 130 = 24x - 120$$

$$-10 = 24x - 25y$$