# 1.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

**In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.**

1. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

| $x$ | 3.9 | 3.99 | 3.999 | 4.001 | 4.01 | 4.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

2. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

3. $\lim_{x \to 0} \frac{\sqrt{x+6}-\sqrt{6}}{x}$

| $x$ | −0.1 | −0.01 | −0.001 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

4. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

| $x$ | −5.1 | −5.01 | −5.001 | −4.999 | −4.99 | −4.9 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

5. $\lim_{x \to 3} \frac{[1/(x+1)]-(1/4)}{x-3}$

| $x$ | 2.9 | 2.99 | 2.999 | 3.001 | 3.01 | 3.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

6. $\lim_{x \to 4} \frac{[x/(x+1)]-(4/5)}{x-4}$

| $x$ | 3.9 | 3.99 | 3.999 | 4.001 | 4.01 | 4.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

7. $\lim_{x \to 0} \frac{\sin x}{x}$

| $x$ | −0.1 | −0.01 | −0.001 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

8. $\lim_{x \to 0} \frac{\cos x - 1}{x}$

| $x$ | −0.1 | −0.01 | −0.001 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

**In Exercises 9–14, create a table of values for the function and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.**

9. $\lim_{x \to 1} \dfrac{x - 2}{x^2 + x - 6}$

10. $\lim_{x \to -3} \dfrac{x + 3}{x^2 + 7x + 12}$

11. $\lim_{x \to 1} \dfrac{x^4 - 1}{x^6 - 1}$

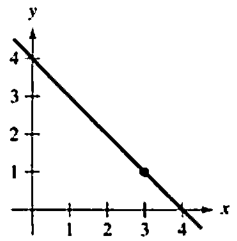
12. $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

13. $\lim_{x \to 0} \dfrac{\sin 2x}{x}$

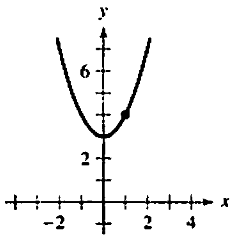
14. $\lim_{x \to 0} \dfrac{\tan x}{\tan 2x}$

**In Exercises 15–24, use the graph to find the limit (if it exists). If the limit does not exist, explain why.**

15. $\lim_{x \to 3} (4 - x)$

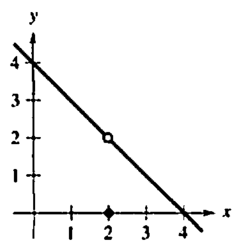



16. $\lim_{x \to 1} (x^2 + 3)$



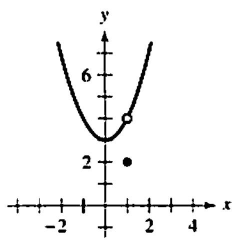

17. $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

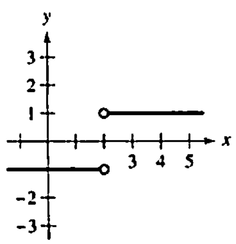



18. $\lim_{x \to 1} f(x)$

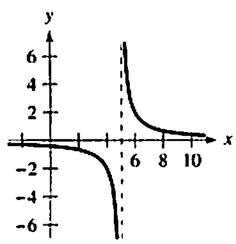
$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



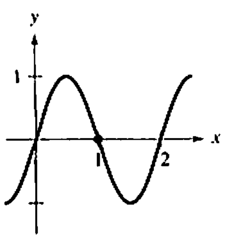

19. $\lim_{x \to 2} \dfrac{|x - 2|}{x - 2}$



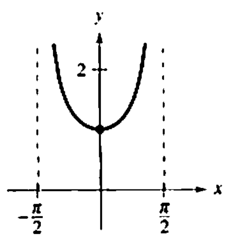

20. $\lim_{x \to 5} \dfrac{2}{x - 5}$



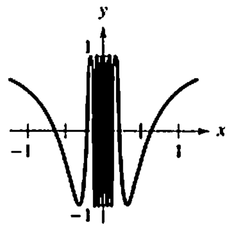

21. $\lim_{x \to 1} \sin \pi x$



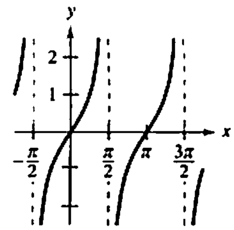

22. $\lim_{x \to 0} \sec x$




23. $\lim_{x \to 0} \cos \dfrac{1}{x}$




24. $\lim_{x \to \pi/2} \tan x$



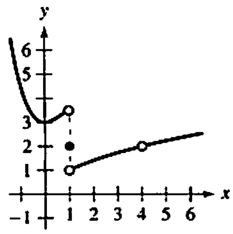

**In Exercises 25 and 26, use the graph of the function $f$ to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.**

25. (a) $f(1)$

(b) $\lim_{x \to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x \to 4} f(x)$




26. (a) $f(-2)$

(b) $\lim_{x \to -2} f(x)$

(c) $f(0)$

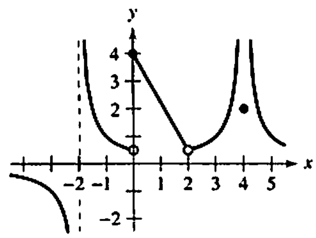
(d) $\lim_{x \to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x \to 2} f(x)$

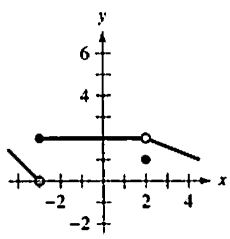
(g) $f(4)$

(h) $\lim_{x \to 4} f(x)$

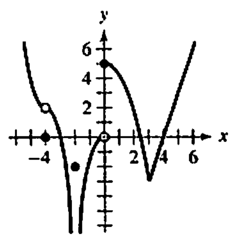



**In Exercises 27 and 28, use the graph of $f$ to identify the values of $c$ for which $\lim_{x \to c} f(x)$ exists.**

27.




28.




**In Exercises 29 and 30, sketch the graph of $f$. Then identify the values of $c$ for which $\lim_{x \to c} f(x)$ exists.**

29. $f(x) = \begin{cases} x^2, & x \leq 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$

30. $f(x) = \begin{cases} \sin x, & x < 0 \\ 1 - \cos x, & 0 \leq x \leq \pi \\ \cos x, & x > \pi \end{cases}$

**In Exercises 31 and 32, sketch a graph of a function $f$ that satisfies the given values. (There are many correct answers.)**

**31.** $f(0)$ is undefined.

$\lim_{x\to 0} f(x) = 4$

$f(2) = 6$

$\lim_{x\to 2} f(x) = 3$

**32.** $f(-2) = 0$

$f(2) = 0$

$\lim_{x\to -2} f(x) = 0$

$\lim_{x\to 2} f(x)$ does not exist.

**33.** ***Modeling Data*** For a long distance phone call, a hotel charges \$9.99 for the first minute and \$0.79 for each additional minute or fraction thereof. A formula for the cost is given by

$C(t) = 9.99 - 0.79[\![-(t-1)]\!]$

where $t$ is the time in minutes.

(*Note:* $[\![x]\!]$ = greatest integer $n$ such that $n \le x$. For example, $[\![3.2]\!] = 3$ and $[\![-1.6]\!] = -2$.)

(a) Use a graphing utility to graph the cost function for $0 < t \le 6$.

(b) Use the graph to complete the table and observe the behavior of the function as $t$ approaches 3.5. Use the graph and the table to find

$\lim_{t\to 3.5} C(t)$.

| $t$ | 3 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 4 |
|---|---|---|---|---|---|---|---|
| $C$ | | | | ? | | | |

(c) Use the graph to complete the table and observe the behavior of the function as $t$ approaches 3.

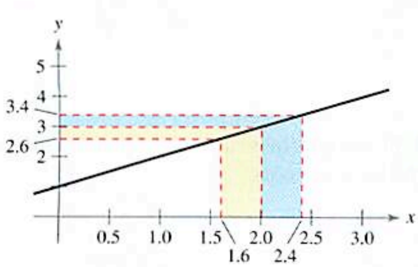
| $t$ | 2 | 2.5 | 2.9 | 3 | 3.1 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| $C$ | | | | ? | | | |

Does the limit of $C(t)$ as $t$ approaches 3 exist? Explain.

**34.** Repeat Exercise 33 for

$C(t) = 5.79 - 0.99[\![-(t-1)]\!]$.

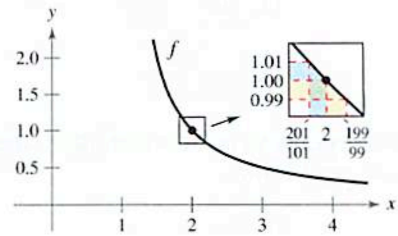
**35.** The graph of $f(x) = x + 1$ is shown in the figure. Find $\delta$ such that if $0 < |x-2| < \delta$ then $|f(x) - 3| < 0.4$.




**36.** The graph of

$$f(x) = \frac{1}{x-1}$$

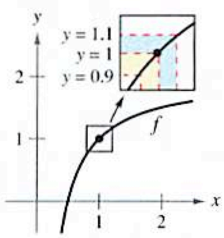
is shown in the figure. Find $\delta$ such that if $0 < |x-2| < \delta$ then $|f(x) - 1| < 0.01$.



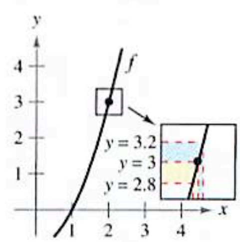

**37.** The graph of

$$f(x) = 2 - \frac{1}{x}$$

is shown in the figure. Find $\delta$ such that if $0 < |x-1| < \delta$ then $|f(x) - 1| < 0.1$.




**38.** The graph of $f(x) = x^2 - 1$ is shown in the figure. Find $\delta$ such that if $0 < |x-2| < \delta$ then $|f(x) - 3| < 0.2$.




**In Exercises 39–42, find the limit $L$. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.**

**39.** $\lim_{x\to 2} (3x + 2)$

**40.** $\lim_{x\to 4} \left(4 - \frac{x}{2}\right)$

**41.** $\lim_{x\to 2} (x^2 - 3)$

**42.** $\lim_{x\to 5} (x^2 + 4)$

*The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.*