M252 Practice Questions for Chapter 14
P. Staley

1. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

   $f(x,y) = 7x^2 + 8xy + 2y^2$

2. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

   $f(x,y) = \sin 9x \cos 3y$

3. Determine whether the vector field is conservative.

   $\vec{\mathbf{F}}(x,y) = 9y^2\left(7y\hat{\mathbf{i}} - x\hat{\mathbf{j}}\right)$

   A) Conservative
   B) Not Conservative

4. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

   $\vec{\mathbf{F}}(x,y) = 7x^6y\hat{\mathbf{i}} + x^7\hat{\mathbf{j}}$

5. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

   $\vec{\mathbf{F}}(x,y) = \frac{6y}{x}\hat{\mathbf{i}} - \frac{x^6}{y^6}\hat{\mathbf{j}}$

6. Find the curl for the vector field at the given point.

   $\vec{\mathbf{F}}(x,y,z) = 2xyz\hat{\mathbf{i}} + 2y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}, \quad (2,3,2)$

7. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

$$\vec{\mathbf{F}}(x,y,z) = 3x^2y^4z^5\hat{\mathbf{i}} + 4x^3y^3z^5\hat{\mathbf{j}} + 5x^3y^4z^4\hat{\mathbf{k}}$$

8. Find the divergence of the vector field.

$$\vec{\mathbf{F}}(x,y,z) = 9x^4\hat{\mathbf{i}} - xy^3\hat{\mathbf{j}}$$

9. Find the divergence of the vector field at the given point.

$$\vec{\mathbf{F}}(x,y,z) = 8xyz\hat{\mathbf{i}} + 8y\hat{\mathbf{j}} + 8x\hat{\mathbf{k}}, \quad (8,9,8)$$

10. Find $\mathbf{curl}\left(\vec{\mathbf{F}} \times \vec{\mathbf{G}}\right)$.

$$\vec{\mathbf{F}}(x,y,z) = 7\hat{\mathbf{i}} + 8x\hat{\mathbf{j}} + 9y\hat{\mathbf{k}}$$
$$\vec{\mathbf{G}}(x,y,z) = 7x\hat{\mathbf{i}} - 7y\hat{\mathbf{j}} + 7z\hat{\mathbf{k}}$$

11. Find $\mathbf{div}\left(\vec{\mathbf{F}} \times \vec{\mathbf{G}}\right)$.

$$\vec{\mathbf{F}}(x,y,z) = 6\hat{\mathbf{i}} + 7x\hat{\mathbf{j}} + 8y\hat{\mathbf{k}}$$
$$\vec{\mathbf{G}}(x,y,z) = 6x\hat{\mathbf{i}} - 6y\hat{\mathbf{j}} + 6z\hat{\mathbf{k}}$$

12. Evaluate the line integral along the given path.

$$\int_C \left(2x - 7y\right) ds \quad C : \vec{\mathbf{r}}(t) = 7t\hat{\mathbf{i}} + 3t\hat{\mathbf{j}}, \quad 0 \le t \le 8$$

13. Evaluate $\int_C \left(x^2 + y^2\right) ds$ where the path C is:

(i) the x-axis from $x = 0$ to $x = 1$;

(ii) the y-axis from $y = 1$ to $y = 3$.

14. Evaluate $\int_C \left(x + 49\sqrt{y}\right) ds$ where the path C is:

(i) the line from (0,0) to (1,1);

(ii) the line from (0,0) to (3,9).

15. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is represented by $\hat{\mathbf{r}}(t)$.

$$\vec{\mathbf{F}}(x, y) = xy\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$
$$C : \vec{\mathbf{r}}(t) = 32t\hat{\mathbf{i}} + t\hat{\mathbf{j}}, \quad 0 \le t \le 8$$

16. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is represented by $\vec{\mathbf{r}}(t)$.

$$\vec{\mathbf{F}}(x, y) = 5x\hat{\mathbf{i}} + 9y\hat{\mathbf{j}}$$
$$C : \vec{\mathbf{r}}(t) = t\hat{\mathbf{i}} + \sqrt{4 - t^2}\,\hat{\mathbf{j}}, \quad -2 \le t \le 2$$

17. Evaluate the line integral along the path C given by $x = 4t$, $y = 20t$, where $0 \le t \le 1$.

$$\int_C \left(x + 4y^2\right) ds$$

18. Evaluate the line integral

$$\int_C (2x - y)\,dx + (x + 3y)\,dy$$

along the path *C*, where *C* is:

(i) the *x*-axis from $x = 0$ to $x = 8$.

(ii) the *y*-axis from $y = 0$ to $y = 12$.

19. Find the area of the lateral surface over the curve *C* in the *xy*-plane and under the surface $z = f(x,y)$, where

Lateral surface area = $\int_C f(x, y)\,ds$

$f(x, y) = 4$, C: line from (0,0) to (5,6).

20. Set up and evaluate the integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ for each parametric representation of *C*.

$$\vec{\mathbf{F}}(x, y) = x^2\hat{\mathbf{i}} + xy\hat{\mathbf{j}}$$

(i) $\vec{\mathbf{r}}_1(t) = 3t\hat{\mathbf{i}} + 7t^2\hat{\mathbf{j}}, \quad 0 \le t \le 1$

(ii) $\vec{\mathbf{r}}_2(t) = 3\sin\theta\hat{\mathbf{i}} + 7\sin^2\theta\hat{\mathbf{j}}, \quad 0 \le \theta \le \dfrac{\pi}{2}$

21. Determine whether or not the vector field is conservative.

$\vec{\mathbf{F}}(x,y) = 40x^7y^7\hat{\mathbf{i}} + 35x^8y^6\hat{\mathbf{j}}$

A) Conservative

B) Not conservative

22. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 2xy\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}, \quad 0 \le t \le 1$

(ii) $\vec{\mathbf{r}}_2(t) = t\hat{\mathbf{i}} + t^7\hat{\mathbf{j}}, \quad 0 \le t \le 1$

23. Find the value of the line integral $\int_C \left(2x - 5y + 7\right)dx - \left(5x + 7y - 8\right)dy$.

(i) C: the curve $x = \sqrt{49 - y^2}$ from (0,–7) to (0,7)

(ii) C: from (0,–7) to (0,7) along $x = \sqrt{49 - y^2}$, then back to (0,–7) along the y-axis (a closed curve).

24. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 3yz\hat{\mathbf{i}} + 3xz\hat{\mathbf{j}} + 3xy\hat{\mathbf{k}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t\hat{\mathbf{k}}, \quad 0 \le t \le 81$

(ii) $\vec{\mathbf{r}}_2(t) = t^2\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t^2\hat{\mathbf{k}}, \quad 0 \le t \le 9$

25. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$$\int_C \left(4y\hat{\mathbf{i}} + 4x\hat{\mathbf{j}}\right) \cdot d\vec{\mathbf{r}}$$

C: a smooth curve from (0,0) to (9,4)

26. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$$\int_C \frac{2x}{\left(x^2+y^2\right)^2}dx + \frac{2y}{\left(x^2+y^2\right)^2}dy$$

C: circle $(x-6)^2+(y-3)^2=49$ clockwise from (13,3) to (–1,3)

27. Verify Green's Theorem by setting up and evaluating both integrals

$$\int_C y^2dx + x^2dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dA$$

for the path C: square with vertices (0,0), (4,0), (4,4), (0,4).

28. Use Green's Theorem to evaluate the integral

$$\int_C (y-x)dx + (5x-y)dy$$

for the path C: boundary of the region lying between the graphs of $y = x$ and $y = x^2 - 18x$

29. Use Green's Theorem to evaluate the integral

$$\int_C 13xydx + (x+y)dy$$

for the path C: boundary of the region lying between the graphs of $y = 0$ and $y = 169 - x^2$.

30. Use Green's Theorem to evaluate the integral

$$\int_C \left(x^2 - y^2\right) dx + 10xydy$$

for the path C: $x^2 + y^2 = 9$.

31. Use Green's Theorem to evaluate the integral

$$\int_C 8xydx + 8(x+y)dy$$

for C: boundary of the region lying between the graphs of $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$.

32. Use Green's Theorem to calculate the work done by the force $\vec{\mathbf{F}}$ on a particle that is moving counterclockwise around the closed path C.

$$\vec{\mathbf{F}}(x, y) = 4xy\hat{\mathbf{i}} + (x+y)\hat{\mathbf{j}}$$
$$C: x^2 + y^2 = 9$$

33. Find the rectangular equation for the surface by eliminating parameters from the vector-valued function. Identify the surface.

$$\vec{\mathbf{r}}(u,v) = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + \frac{v}{9}\hat{\mathbf{k}}$$

34. Find a vector-valued function whose graph is the cylinder $x^2 + y^2 = 4$.

35. Find a vector-valued function whose graph is the ellipsoid $\frac{x^2}{49} + \frac{y^2}{100} + \frac{z^2}{81} = 1$.

36. Write a set of parametric equations for the surface of revolution obtained by revolving the graph of the function about the given axis.

$$y = \frac{x}{7}, \quad 0 \le x \le 21 \qquad x\text{-axis}$$

37. Find an equation of the tangent plane to the surface represented by the vector-valued function at the given point.

$$\vec{\mathbf{r}}(u,v) = (10u+v)\hat{\mathbf{i}} + (u-v)\hat{\mathbf{j}} + v\hat{\mathbf{k}}, \quad (6,-6,6)$$

38. Find the area of the surface over the given region. Use a computer algebra system to verify your results.

The sphere,

$$\vec{\mathbf{r}}(u,v) = 8\sin u\cos v\hat{\mathbf{i}} + 8\sin u\sin v\hat{\mathbf{j}} + 8\cos u\hat{\mathbf{k}}, \quad 0 \le u \le \pi, \quad 0 \le v \le 2\pi$$

39. Find the area of the surface over the given region. Use a computer algebra system to verify your results.

The part of the cone,

$$\vec{\mathbf{r}}(u,v) = 3u\cos v\hat{\mathbf{i}} + 3u\sin v\hat{\mathbf{j}} + u\hat{\mathbf{k}}$$

where $0 \le u \le 8$ and $0 \le v \le 2\pi$.

40. Evaluate $\iint\limits_S (x-7y+z)\,dS$, where

$$S\colon\ z = 14 - x, \quad 0 \le x \le 14, \quad 0 \le y \le 14$$

41. Evaluate $\iint\limits_S f(x,y)\,dS$, where

$$f(x,y) = y+6$$

$$S: r(u,v) = u\mathbf{i} + v\mathbf{j} + \frac{v}{10}\mathbf{k}, \quad 0 \le u \le 1, \quad 0 \le v \le 10$$

42. Evaluate $\iint\limits_S f(x,y,z)\,dS$, where

$$f(x,y,z) = x^2 + y^2 + z^2$$

$$S: x^2 + y^2 = 4, \quad 0 \le z \le 4$$

43. Find the flux of $\vec{\mathbf{F}}$ through *S*, $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$, where $\vec{\mathbf{N}}$ is the upward unit normal vector to *S*.

$\vec{\mathbf{F}}(x, y, z) = 4z\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + y\hat{\mathbf{k}}$
$S : x + y + z = 10,$ first octant

44. Find the flux of $\vec{\mathbf{F}}$ through *S*, $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$, where $\vec{\mathbf{N}}$ is the upward unit normal vector to *S*.

$\vec{\mathbf{F}}(x, y, z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
$S : z = 36 - x^2 - y^2, \quad z \geq 0$

45. Find the flux of $\vec{\mathbf{F}}$ over the closed surface (let $\vec{\mathbf{N}}$ be the outward unit normal vector of the surface).

$\vec{\mathbf{F}}(x, y, z) = 49xy\hat{\mathbf{i}} + z^2\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$
$S :$ cube bounded by $x = 0, x = 6, y = 0, y = 6, z = 0, z = 6$

46. Use the Divergence Theorem to evaluate $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$. Verify your answer by evaluating the integral as a triple integral.

$F(x, y, z) = 2x\hat{\mathbf{i}} - 2y\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$
$S :$ cube bounded by the planes $x = 0, x = 8, y = 0, y = 8, z = 0, z = 8$

47. Use the Divergence Theorem to evaluate $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$ and find the outward flux of $\vec{\mathbf{F}}$ through the surface of the solid bounded by the graphs of the equations. Use a computer algebra system to verify your results.

$\vec{\mathbf{F}}(x, y, z) = x^2\hat{\mathbf{i}} - 2xy\hat{\mathbf{j}} + xyz^2\hat{\mathbf{k}}$
$S : \; z = \sqrt{100 - x^2 - y^2}, \; z = 0$

48. Use the Divergence Theorem to evaluate $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$ and find the outward flux of $\vec{\mathbf{F}}$ through the surface of the solid bounded by the graphs of the equations. Use a computer algebra system to verify your results.

$\vec{\mathbf{F}}(x, y, z) = xyz\hat{\mathbf{j}}$
$S: \ x^2 + y^2 = 25, \ z = 0, \ z = 7$

49. Use the Divergence Theorem to evaluate $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$ and find the outward flux of $\vec{\mathbf{F}}$ through the surface of the solid bounded by the graphs of the equations. Use a computer algebra system to verify your results.

$\vec{\mathbf{F}}(x, y, z) = xy\hat{\mathbf{i}} + 100y\hat{\mathbf{j}} + xz\hat{\mathbf{k}}$
$S: \ x^2 + y^2 + z^2 = 100$

50. Evaluate $\iint_S \mathbf{curl}\vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$ where S is the closed surface of the solid bounded by the graphs of $z = 16 - y^2$, $z = 0$, $x = 0$, and $x = 25$.

$\vec{\mathbf{F}}(x, y, z) = xy\cos z\hat{\mathbf{i}} + yz\sin x\hat{\mathbf{j}} + xyz\hat{\mathbf{k}}$

51. Find the curl of the vector field $\vec{\mathbf{F}}(x, y, z) = (10y - z)\hat{\mathbf{i}} + xyz\hat{\mathbf{j}} + 6e^z\hat{\mathbf{k}}$

52. Use Stokes's Theorem to evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. Use a computer algebra system to verify your results. Note: C is oriented counterclockwise as viewed from above.

$\vec{\mathbf{F}}(x, y, z) = 6y\hat{\mathbf{i}} + 12z\hat{\mathbf{j}} + x\hat{\mathbf{k}}$
$C:$ triangle with vertices $(0,0,0), (0,6,0), (1,1,1)$

53. Use Stokes's Theorem to evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. Use a computer algebra system to verify your results. Note: C is oriented counterclockwise as viewed from above.

$\vec{\mathbf{F}}(x, y, z) = 36xz\hat{\mathbf{i}} + y\hat{\mathbf{j}} + 36xy\hat{\mathbf{k}}$
$S: \ z = 100 - x^2 - y^2, z \geq 0$