M252 Practice Questions for Chapter 14

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1. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

   $$f(x,y) = 7x^2 + 8xy + 2y^2$$

2. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

   $$f(x,y) = \sin 9x \cos 3y$$

3. Determine whether the vector field is conservative.

   $$\vec{\mathbf{F}}(x,y) = 9y^2\left(7y\hat{\mathbf{i}} - x\hat{\mathbf{j}}\right)$$

   A) Conservative
   B) Not Conservative

4. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

   $$\vec{\mathbf{F}}(x,y) = 7x^6 y\hat{\mathbf{i}} + x^7\hat{\mathbf{j}}$$

5. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

   $$\vec{\mathbf{F}}(x,y) = \frac{6y}{x}\hat{\mathbf{i}} - \frac{x^6}{y^6}\hat{\mathbf{j}}$$

6. Find the curl for the vector field at the given point.

   $$\vec{\mathbf{F}}(x,y,z) = 2xyz\hat{\mathbf{i}} + 2y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}, \quad (2,3,2)$$

7. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

$$\vec{\mathbf{F}}(x,y,z)=3x^2y^4z^5\hat{\mathbf{i}}+4x^3y^3z^5\hat{\mathbf{j}}+5x^3y^4z^4\hat{\mathbf{k}}$$

8. Find the divergence of the vector field.

$$\vec{\mathbf{F}}(x,y,z)=9x^4\hat{\mathbf{i}}-xy^3\hat{\mathbf{j}}$$

9. Find the divergence of the vector field at the given point.

$$\vec{\mathbf{F}}(x,y,z)=8xyz\hat{\mathbf{i}}+8y\hat{\mathbf{j}}+8x\hat{\mathbf{k}},\quad (8,9,8)$$

12. Evaluate the line integral along the given path.

$$\int_C (2x-7y)\,ds \quad C:\vec{\mathbf{r}}(t)=7t\hat{\mathbf{i}}+3t\hat{\mathbf{j}},\quad 0\le t\le 8$$

13. Evaluate $\int_C \left(x^2 + y^2\right) ds$ where the path C is:

(i) the x-axis from $x = 0$ to $x = 1$;

(ii) the y-axis from $y = 1$ to $y = 3$.

14. Evaluate $\int_C \left(x + 49\sqrt{y}\right) ds$ where the path C is:

(i) the line from (0,0) to (1,1);

(ii) the line from (0,0) to (3,9).

15. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is represented by $\hat{\mathbf{r}}(t)$.

$$\vec{\mathbf{F}}(x, y) = xy\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$
$$C : \vec{\mathbf{r}}(t) = 32t\hat{\mathbf{i}} + t\hat{\mathbf{j}}, \quad 0 \le t \le 8$$

16. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is represented by $\vec{\mathbf{r}}(t)$.

$$\vec{\mathbf{F}}(x, y) = 5x\hat{\mathbf{i}} + 9y\hat{\mathbf{j}}$$
$$C : \vec{\mathbf{r}}(t) = t\hat{\mathbf{i}} + \sqrt{4 - t^2}\,\hat{\mathbf{j}}, \quad -2 \le t \le 2$$

17. Evaluate the line integral along the path C given by $x = 4t$, $y = 20t$, where $0 \le t \le 1$.

$$\int_C \left(x + 4y^2\right) ds$$

18. Evaluate the line integral

$$\int_C (2x - y)\,dx + (x + 3y)\,dy$$

along the path *C*, where *C* is:

(i) the *x*-axis from $x = 0$ to $x = 8$.

(ii) the *y*-axis from $y = 0$ to $y = 12$.

19. Find the area of the lateral surface over the curve *C* in the *xy*-plane and under the surface $z = f(x,y)$, where

Lateral surface area = $\int_C f(x, y)\,ds$

$f(x, y) = 4$, C: line from (0,0) to (5,6).

20. Set up and evaluate the integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ for each parametric representation of *C*.

$$\vec{\mathbf{F}}(x, y) = x^2\hat{\mathbf{i}} + xy\hat{\mathbf{j}}$$

(i) $\vec{\mathbf{r}}_1(t) = 3t\hat{\mathbf{i}} + 7t^2\hat{\mathbf{j}}, \quad 0 \le t \le 1$

(ii) $\vec{\mathbf{r}}_2(t) = 3\sin\theta\hat{\mathbf{i}} + 7\sin^2\theta\hat{\mathbf{j}}, \quad 0 \le \theta \le \frac{\pi}{2}$

21. Determine whether or not the vector field is conservative.

$\vec{\mathbf{F}}(x,y) = 40x^7y^7\hat{\mathbf{i}} + 35x^8y^6\hat{\mathbf{j}}$

A) Conservative

B) Not conservative

22. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 2xy\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}, \quad 0 \le t \le 1$

(ii) $\vec{\mathbf{r}}_2(t) = t\hat{\mathbf{i}} + t^7\hat{\mathbf{j}}, \quad 0 \le t \le 1$

23. Find the value of the line integral $\int_C (2x - 5y + 7)\,dx - (5x + 7y - 8)\,dy$.

(i) C: the curve $x = \sqrt{49 - y^2}$ from (0,–7) to (0,7)

(ii) C: from (0,–7) to (0,7) along $x = \sqrt{49 - y^2}$, then back to (0,–7) along the y-axis (a closed curve).

24. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 3yz\hat{\mathbf{i}} + 3xz\hat{\mathbf{j}} + 3xy\hat{\mathbf{k}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t\hat{\mathbf{k}}, \quad 0 \le t \le 81$

(ii) $\vec{\mathbf{r}}_2(t) = t^2\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t^2\hat{\mathbf{k}}, \quad 0 \le t \le 9$

25. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$$\int_C \left(4y\hat{\mathbf{i}} + 4x\hat{\mathbf{j}}\right) \cdot d\vec{\mathbf{r}}$$

C: a smooth curve from (0,0) to (9,4)

26. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$$\int_C \frac{2x}{\left(x^2+y^2\right)^2}\,dx + \frac{2y}{\left(x^2+y^2\right)^2}\,dy$$

C: circle $(x-6)^2 + (y-3)^2 = 49$ clockwise from (13,3) to (–1,3)

28. Use Green's Theorem to evaluate the integral

$$\int_C (y-x)dx + (5x-y)dy$$

for the path *C*: boundary of the region lying between the graphs of $y = x$ and $y = x^2 - 18x$

29. Use Green's Theorem to evaluate the integral

$$\int_C 13xydx + (x+y)dy$$

for the path *C*: boundary of the region lying between the graphs of $y = 0$ and $y = 169 - x^2$.

30. Use Green's Theorem to evaluate the integral

$$\int_C \left(x^2 - y^2\right) dx + 10xy dy$$

for the path C: $x^2 + y^2 = 9$.

31. Use Green's Theorem to evaluate the integral

$$\int_C 8xy dx + 8(x + y) dy$$

for C: boundary of the region lying between the graphs of $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$.

32. Use Green's Theorem to calculate the work done by the force $\vec{\mathbf{F}}$ on a particle that is moving counterclockwise around the closed path C.

$$\vec{\mathbf{F}}(x, y) = 4xy\hat{\mathbf{i}} + (x + y)\hat{\mathbf{j}}$$
$$C: x^2 + y^2 = 9$$

33. Find the rectangular equation for the surface by eliminating parameters from the vector-valued function. Identify the surface.

$$\vec{\mathbf{r}}(u, v) = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + \frac{v}{9}\hat{\mathbf{k}}$$