1. Find and simplify the function values.

$f(x,y) = 5 - x^2 - 10y^2$

(i) $f(0,0)$ (ii) $f(0,1)$ (iii) $f(3,9)$

(iv) $f(1,y)$ (v) $f(x,0)$ (vi) $f(t,1)$

2. Find and simplify the function values.

$h(x,y,z) = \dfrac{xy}{z}$

(i) $h(7,8,24)$ (ii) $h(6,5,6)$ (iii) $h(-7,8,9)$ (iv) $h(10,9,-16)$

3. Find and simplify the function values.

$g(x,y) = \int_x^y (16t - 6)dt$

(i) $g(0,32)$ (ii) g(1,32) (iii) $g\left(\dfrac{3}{8},32\right)$ (iv) $g\left(0,\dfrac{3}{8}\right)$

4. Describe the domain and range of the function.

$f(x,y) = \sqrt{100 - x^2 - y^2}$

5. Describe the level curves of the function. Sketch the level curves for the given *c*-values.

$z = 6 - 2x - 3y$, $c = 0, 2, 4, 6$

6. Find the limit and discusss the continuity of the function.

$$\lim_{(x,y)\to(-5,4)} \left(x + 7y^2\right)$$

7. Find the limit and discusss the continuity of the function.

$$\lim_{(x,y)\to(2,-10)} \left(8x + y + 2\right)$$

8. Find the limit and discusss the continuity of the function.

$$\lim_{(x,y)\to(1,1)} \frac{x}{\sqrt{7x+6y}}$$

9. Find the limit and discusss the continuity of the function.

$$\lim_{(x,y,z)\to(-1,0,8)} 5xe^{y^6 z}$$

10. Find the limit (if it exists). If the limit does not exist, explain why.

$$\lim_{(x,y)\to(4,3)} \frac{xy-4}{3+xy}$$

11. Discusss the continuity of the function at the origin.

$$f(x,y) = \begin{cases} \dfrac{4x^8y^8}{x^8+y^8}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$$

12. Use polar coordinates to find the limit. [Hint: Let $x = r\cos\theta$ and $y = r\sin\theta$, and note that $(x,y) \to (0,0)$ implies $r \to 0$.]

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$$

13. Discusss the continuity of the function.

$$f(x,y,z) = \frac{z}{x^2+y^2-64}$$

14. Find each limit for the function $f(x, y) = -7x^2 - 4y$.

(i) $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$

(ii) $\lim\limits_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) - f(x, y)}{\Delta y}$

15. Find both first partial derivatives.

$f(x, y) = -4x + 3y - 8$

16. Find both first partial derivatives.

$f(x, y) = -4x^2 + 5y^2 + 1$

17. Find both first partial derivatives.

$z = x \cdot \sqrt[4]{y}$

18. Find both first partial derivatives.

$z = 2y^3\sqrt{x}$

19. Find both first partial derivatives.

$$z = x^6 e^{10y}$$

20. Find both first partial derivatives.

$$f(x, y) = \ln\left(x^5 + y^8\right)$$

21. Find both first partial derivatives.

$$f(x, y) = \ln \sqrt[7]{xy}$$

22. Find both first partial derivatives.

$$z = \frac{x^2}{9y} + \frac{5y^2}{x}$$

23. Find both first partial derivatives.

$$f(x, y) = \sqrt{4x^{10} + y^4}$$

24. Evaluate f_x and f_y at the given point.

$$f(x, y) = \frac{3xy}{\sqrt{9x^2 + 2y^2}}, \quad (1,1)$$

25. For $f(x,y)$, find all values of x and y such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously.

$$f(x, y) = 9x^3 - 3xy + 9y^3$$

26. Find the first partial derivatives with respect to x, y, and z.

$$w = \frac{2xz}{9x + 6y}$$

27. Find the first partial derivatives with respect to x, y, and z.

$$H(x, y, z) = \cos(2x + 8y + 7z)$$

28. Evaluate f_x and f_y at the given point.

$$f(x, y, z) = \frac{xy}{x + y + z}, \quad (6,8,3)$$

29. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = x^2 + 2xy + 8y^2$$

30. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = 11xe^y + 8ye^{-x}$$

31. Find the total differential of the function $z = 5x^{10}y^9$.

32. Find the total differential of the function $z = \dfrac{x^9}{y}$.

33. Find the total differential of the function $z = -\dfrac{1}{x^7 + y^5}$.

34. For the function $f(x, y) = 5x - 4y$:

(i) Evaluate *f*(1,5) and *f*(1.09,5.09) and calculate Δz, and

(ii) Use the total differential *dz* to approximate Δz.

35. For the function $f(x, y) = x \sin y$:

(i) Evaluate $f(4,2)$ and $f(4.08,2.03)$ and calculate Δz, and

(ii) Use the total differential dz to approximate Δz.

36. The radius r and height h of a right circular cylinder are measured with possible errors of 8% and 3%, respectively. Approximate the maximum possible percent error in measuring the volume.

37. A triangle is measured and two adjacent sides are found to be 3 inches and 4 inches long, with an included angle of $\frac{\pi}{4}$. The possible errors in measurement are $\frac{1}{10}$ inch for the sides and 0.04 radian for the angle. Approximate the maximum possible error in the computation of the area.

38. Let $w = xy$, where $x = 10 \sin t$ and $y = -8 \cos t$. Find $\frac{dw}{dt}$.

39. Let $w = \cos(2x - 4y)$, where $x = t^9$ and $y = 7$. Find $\frac{dw}{dt}$.

40. Let $w = xy\cos z$, where $x = t^9$, $y = t^2$, and $z = \arccos t$. Find $\dfrac{dw}{dt}$.

41. Let $w = x^3 + y^3$, where $x = 4s + t$, $y = 4s - t$. Find $\dfrac{\partial w}{ds}$ and $\dfrac{\partial w}{dt}$ and evaluate each partial derivative at the point $s = 2$, $t = 2$.

42. Let $w = x^7 - 7x^6 y$, where $x = e^s$, $y = e^t$. Find $\dfrac{\partial w}{ds}$ and $\dfrac{\partial w}{dt}$ and evaluate each partial derivative at the point $s = 0$, $t = 1$.

43. Let $w = (x - y)^3$, where $x = r + \theta$ and $y = r - \theta$. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.

44. Let $w = \dfrac{yz}{x}$, where $x = \theta^2$, $y = 9r + 7\theta$, and $z = 9r - 7\theta$. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.

45. Differentiate implicitly to find $\dfrac{dy}{dx}$.

$$x^2 - 9xy + y^2 - 6x + y - 7 = 0$$

46. Differentiate implicitly to find the first partial derivatives of z.

$$x^9 + y^9 + z^9 = 8$$

47. Differentiate implicitly to find the first partial derivatives of z.

$$x^3 + \sin(6y + 7z) = 0$$

48. Differentiate implicitly to find the first partial derivatives of w.

$$x^6 + y^6 + z^6 - 7yw + 9w^{10} = 10$$

49. The radius of a right cylinder is increasing at a rate of 2 inches per minute, and the height is decreasing at a rate of 3 inches per minute. What is the rate of change of the volume and surface area (including both ends of the cylinder as well as the side) when the radius is 9 inches and height is 27 inches?

50. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x, y) = 5x - 7xy + 10y, \quad P(1, 4), \quad \vec{\mathbf{v}} = \frac{1}{2}\left(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}\right)$$

51. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x,y) = x^3 - y^3, \quad P(2,1), \quad \vec{\mathbf{v}} = \frac{\sqrt{2}}{2}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

52. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x,y,z) = xy + yz + xz, \quad P(1,1,1), \quad \vec{\mathbf{v}} = 9\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$$

53. Find the directional derivative of the function in the direction of $\vec{\mathbf{u}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$.

$$f(x,y) = \sin(3x - 2y), \quad \theta = \frac{\pi}{3}$$

54. Find the gradient of the function at the given point.

$$f(x,y) = 9x - 4y^2 + 2, \quad (4,1)$$

55. Find the gradient of the function at the given point.

$$g(x,y) = 9xe^{\frac{y}{x}}, \quad (9,0)$$

56. Find the gradient of the function at the given point.

$w = 6x^2 y - 10yz + z^2, \quad (1,1,-2)$

57. Use the gradient to find the directional derivative of the function at *P* in the direction of *Q*.

$g(x, y) = x^2 + y^2 + 1, \quad P(2,4) \quad Q(6,12)$

58. Use the gradient to find the directional derivative of the function at *P* in the direction of *Q*.

$f(x, y) = \sin(7x)\cos y, \quad P(0,0), \quad Q(\frac{\pi}{7}, \pi)$

59. Find the gradient of the function and the maximum value of the directional derivative at the given point.

$w = xy^4 z^7, \quad (9,1,1)$

60. Find the directional derivative $D_{\mathrm{u}} f(3,2)$ of the function $f(x, y) = 10 - \dfrac{x}{10} - \dfrac{y}{3}$ in the direction of $\vec{\mathbf{u}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$.

(i) $\theta = \dfrac{\pi}{4}$; (ii) $\theta = \dfrac{2\pi}{3}$

61. Find the directional derivative $D_{\mathbf{u}} f(3,2)$ of the function $f(x,y) = 10 - \frac{x}{10} - \frac{y}{5}$ in the direction of $\vec{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{\|\vec{\mathbf{v}}\|}$.

(i) $\vec{\mathbf{v}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$; (ii) $\vec{\mathbf{v}} = -3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

62. Find the directional derivative $D_{\mathbf{u}} f(3,2)$ of the function $f(x,y) = 10 - \frac{x}{10} - \frac{y}{5}$ in the direction of $\vec{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{\|\vec{\mathbf{v}}\|}$.

(i) $\vec{\mathbf{v}}$ is the vector from (1,2) to (–2 ,6); (ii) $\vec{\mathbf{v}}$ is the vector from (3,2) to (4,5).

63. Find $\nabla f(x,y)$ for function $f(x,y) = 5 - \frac{x}{5} - \frac{y}{8}$.

64. For function $f(x,y) = 6 - \frac{x}{6} - \frac{y}{9}$, find the maximum value of the directional derivative at (3,2).

65. Use the gradient to find a normal vector to the graph of the equation at the given point.

$9x^2 - y = 4, \quad (10, 896)$

66. Find a unit normal vector to the surface $x + y + z = 6$ at the point (3,0,3).

67. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 18$ at the point (4,1,1).

68. Find a unit normal vector to the surface $x^4 y^2 - z = 0$ at the point $(1, 6, 36)$.

69. Find a unit normal vector to the surface $x^2 + 2y + z^3 = 10$ at the point $(2, -1, 2)$.

70. Find an equation of the tangent plane to the surface $g(x, y) = x^2 - y^2$ at the point $(4, 6, -20)$.

71. Find an equation of the tangent plane to the surface $f(x, y) = 10 - \frac{5}{4}x - y$ at the point $(8, -1, 1)$.

72. Find an equation of the tangent plane to the surface $x^2 + 9y^2 + z^2 = 504$ at the point $(6, -6, 12)$.

73. Find an equation of the tangent plane to the surface $x = y(7z - 5)$ at the point $(96, 6, 3)$.

74. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $x^2 + y^2 + z^2 = 201$ at the point $(1, 10, 10)$.

75. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xy - 4z = 0$ at the point $(-4, -10, 10)$.

76. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xyz = 30$ at the point $(1, 3, 10)$.

77. Find the angle of inclination θ of the tangent plane to the surface $x^2 - y^2 + z = 0$ at the point $(1, 3, 8)$.

78. Find the angle of inclination θ of the tangent plane to the surface $3x^2 + 4y^2 - z = 0$ at the point $(6, 6, 252)$.

79. Find the distance from the point (0,0,0) to the plane $9x + 10y + z = 5$.

80. Find three positive numbers *x*, *y*, and *z* whose sum is 3 and product is a maximum.

81. Find three positive numbers *x*, *y*, and *z* whose sum is 24 and the sum of the squares is a maximum.

82. The sum of the length (denote by *z*) and the girth (perimeter of a cross section) of packages carried by a delivery service cannot exceed 36 inches. Find the dimensions of the rectangular package of largest volume that may be sent.

83. **OMIT**

84. A company manufactures two types of sneakers: running shoes and basketball shoes. The total revenue from x_1 units of running shoes and y_1 units of basketball shoes is:

$$R = -3x_1^2 - 8x_2^2 - 2x_1x_2 + 40x_1 + 109x_2 \ ,$$

where x_1 and x_2 are in thousands of units. Find x_1 and x_2 so as to maximize the revenue.

85. Find the least squares regression line for the points (1,0) , (3,3) , (10,6).