M252 Practice exam 5 sections 13.1-13.3, 13.5-13.7

1. Evaluate the following integral.

$$\int_{9}^{4y} \frac{6y}{x}\,dx$$

2. Evaluate the following integral.

$$\int_{4x}^{x^4} \frac{-10y}{x}\,dy$$

3. Evaluate the following integral.

$$\int_{y}^{\frac{\pi}{11}} \sin^3 3x \cos y\,dx$$

4. Evaluate the following iterated integral.

$$\int_{2}^{6}\int_{1}^{6}\left(4x+y\right)\,dy\,dx$$

5. Evaluate the following iterated integral.

$$\int_{6}^{7}\int_{1}^{\sqrt{x}} 2ye^{-x}\ dy\,dx$$

6. Evaluate the following iterated integral.

$$\int_{3}^{5}\int_{y}^{-2y} \left(-2+6x^2+6y^2\right)\,dx\,dy$$

7. Evaluate the following iterated integral.

$$\int_{0}^{\frac{\pi}{9}}\int_{0}^{11\cos\theta} r\ dr\,d\theta$$

8. Evaluate the following improper integral.

$$\int_{2}^{\infty}\int_{0}^{7/x} y\ dy\,dx$$

9. Use an iterated integral to find the area of the region bounded by

$\sqrt{x}+\sqrt{y}=2, \quad x=0, \quad y=0$.

10. Use an iterated integral to find the area of the region bounded by

$4x-3y=0, \quad x+y=2, \quad y=0$.

11. Sketch the region, *R*, of integration and then switch the order of integration for the following integral.

$$\int_0^4 \int_0^{\sqrt{16-x^2}} f(x,y)\,dydx$$

12. Sketch the region, *R*, whose area is given by the iterated integral below. Then switch the order of integration and evaluate the integral.

$$\int_{-9}^{9} \int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} dydx$$

13. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^5 \int_x^5 e^{-8y^2} \ dydx$$

14. Sketch the region R and evaluate the iterated integral.

$$I = \int_0^6 \int_{x/2}^3 (x+y)dydx$$

15. Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral below over the region R.

$$\iint_R \frac{y}{x^2+y^2} dA$$

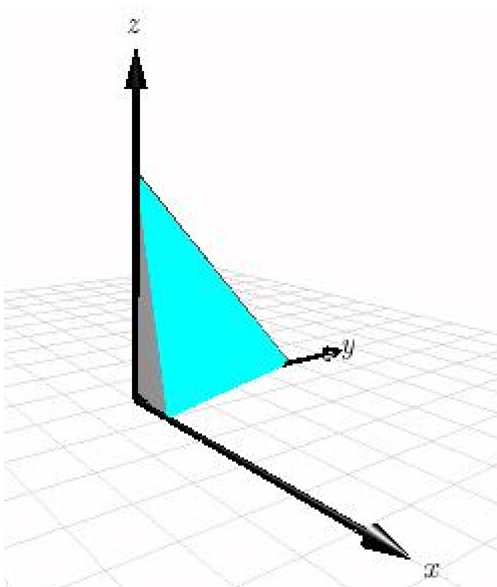
R : triangle bounded by $y = 5x,\ y = 8x,$ and $x = 5$

16. Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral below over the region R.

$$\iint_R -3y \ln x \, dA$$

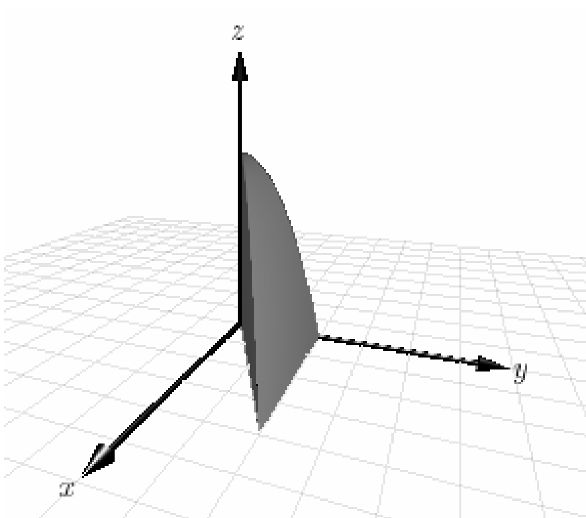
R : region bounded by $y = 1 - x$ and $y = 1 - x^2$

17. Use a double integral to find the volume of the indicated solid.



$4x + y + z = 2, \ \ x > 0, y > 0, z > 0$

18. Use a double integral to find the volume of the indicated solid.



$z = 4 - y^2, \ \ z > 0, \ x > 0, \ 3x < y < 2$

19. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$z = xy$, $z > 0$, $x > 0$, $7x < y < 4$

20. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$z = \frac{1}{25 + y^2}$, $x = 0$, $x = 2$, $y \geq 0$

21. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^5 \int_x^5 e^{-0.29y^2} \, dydx$$

22. Find the average value of $f(x,y)$ over the region R where:

Average value = $\frac{1}{A}\iint_R f(x, y)dA$

$f(x, y) = xy$

R: rectangle with vertices $(0,0)$, $(1,0)$, $(1,5)$, $(0,5)$

23. Identify the region of integration for the following integral.

$$\int_0^{\pi/2}\int_0^{1+\sin\theta} r\theta\, dr\, d\theta$$

24. Evaluate the double integral below.

$$\int_0^{2\pi}\int_0^{6} 5\, r^5 \sin\theta\, dr\, d\theta$$

25. Evaluate the double integral below.

$$\int_0^{\pi/7}\int_0^{3} r^3 \sin\theta\cos\theta\ dr\, d\theta$$

26. Evaluate the double integral below.

$$\int_0^{\pi/2}\int_0^{5+3\sin\theta} \theta r\ dr\, d\theta$$

27. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} y\, dy\, dx$$

28. Evaluate the following iterated integral by converting to polar coodinates.

$$\int_0^4 \int_0^{\sqrt{6x-x^2}} \frac{1}{5} xy \; dydx$$

29. Combine the sum of the two iterated integrals into a single integral by converting to polar coordinates. Evaluate the resulting iterated integral.

$$\int_0^7 \int_0^x \sqrt{x^2+y^2} \; dydx + \int_7^{7\sqrt{2}} \int_0^{\sqrt{98-x^2}} \sqrt{x^2+y^2} \; dydx$$

30. Use a double integral in polar coodinates to find the volume of the solid in the first octant bounded by the graphs of the equations given below.

$$z = x^4 y, \quad x^2 + y^2 = 16$$

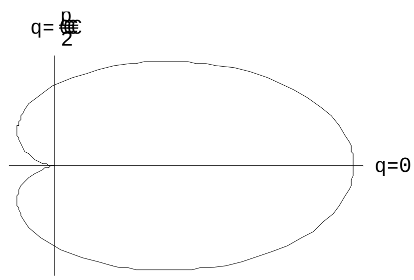
31. Use a double integral in polar coodinates to find the volume of the solid inside the hemisphere

$$z=\sqrt{81-x^2-y^2}$$

but outside the cylinder

$$x^2+y^2=49 .$$

32. Use a double integral to find the area enclosed by the graph of $r=5+5\cos\theta$.



33. Use a double integral to find the area enclosed by the graph of $r=7\cos 7\theta$.

34. Find the area of the surface given by $z=f(x,y)$ over the region R.

$$f(x,y)=-6-4x+2y$$

R: square with vertices $(0,0)$, $(4,0)$, $(4,4)$, $(0,4)$

35. Find the area of the surface given by $z = f(x,y)$ over the region R.

$$f(x, y) = 10 + 2x - 4y$$

R: rectangle with vertices $(0,0)$, $(4, 0)$, $(4, 5)$, $(0, 5)$

36. Find the area of the surface given by $z = f(x,y)$ over the region R.

$$f(x, y) = -1 - x^2$$

R: rectangle with vertices $(0,0)$, $(7, 0)$, $(7, 7)$, $(0, 7)$

37. Find the area of the surface given by $z = f(x,y)$ over the region R.

$$f(x, y) = xy$$

R: $\{(x, y) : x^2 + y^2 \leq 121\}$

38. Find the area of the surface of the portion of the plane

$$z = 5 - 2x - 8y$$

in the first octant.

39. Set up a double integral that gives the area of the surface of the graph of *f* over the region *R*.

$$f(x, y) = x^4 - 3xy + 7y^3$$

$$R = \{(x, y) : -5 \le x \le 5, -2 \le y \le 2\}$$

40. Set up a double integral that gives the area of the surface of the graph of *f* over the region *R*.

$$f(x, y) = e^{7xy}$$

$$R = \{(x, y) : 0 \le x \le 2, 0 \le y \le 4\}$$

41. Evaluate the following iterated integral.

$$\int_0^6 \int_0^4 \int_0^5 (5x + 3y + z)\, dx\, dy\, dz$$

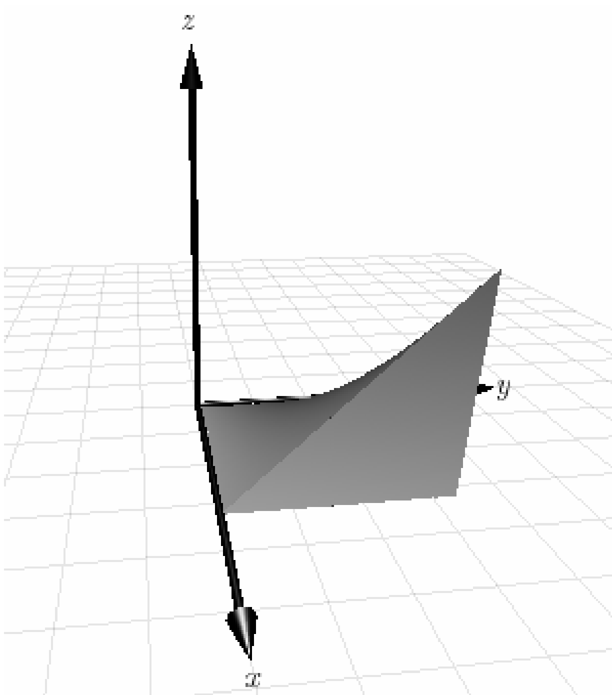
42. Evaluate the following iterated integral.

$$\int_1^8 \int_0^1 \int_0^x 2ze^{-x^2}\, dy\, dx\, dz$$

43. Set up a triple integral for the volume of the solid bounded by the coordinate planes and the plane given below.

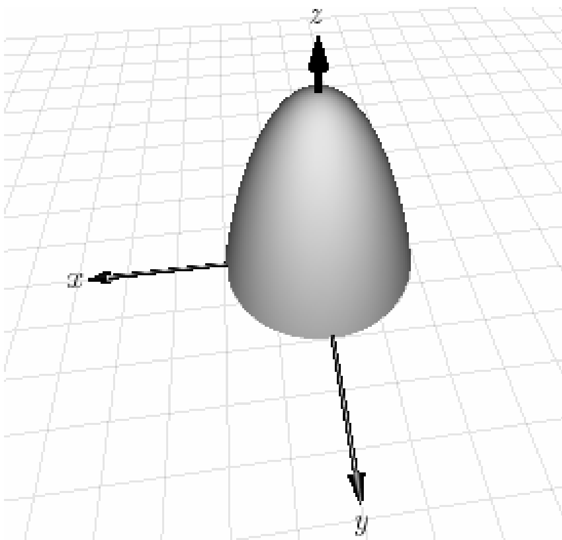
$z = 6 - 2x - 4y$

44. Use a triple integral for the volume of the solid shown below.



$z = 12xy, \quad 0 \le x \le 5, \quad 0 \le y \le 6$

45. Use a triple integral for the volume of the solid shown below.



$z = 64 - x^2 - y^2, \quad z \geq 0$

46. Sketch the solid whose volume is given by the iterated integral given below and re-write the integral using the indicated order of integration.

$$\int_0^3 \int_y^3 \int_0^{\sqrt{9-y^2}} dzdxdy$$

Rewrite the integral using the order *dzdydx*.

47. Find the average value of the function *f* over the region in the first octant bounded by the coordinate planes, and the planes $x = 4$, $y = 3$, and $z = 2$.

$$f(x, y, z) = x^3 + y^2 + z^4$$

48. Evaluate the following iterated integral.

$$\int_0^{\pi/2} \int_0^{1\cos^4\theta} \int_0^{1-r^2} r \sin\theta \; dz\, dr\, d\theta$$

49. Evaluate the following iterated integral.

$$\int_0^{\pi/13} \int_0^{\pi/13} \int_0^{\cos\theta} \rho^2 \sin\phi \cos\phi \; d\rho\, d\theta\, d\phi$$

50. Evaluate the following iterated integral.

$$\int_0^{2\pi} \int_0^{\pi} \int_3^{5} \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

51. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{9} x \; dz\, dy\, dx$$

52. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \sqrt{x^2+y^2+z^2}\ dz\,dy\,dx$$

53. Use cylindrical coordinates to find the volume inside the sphere

$$x^2+y^2+z^2=4$$

and above the upper nappe of the cone $z^2=\left(x^2+y^2\right)$.

54. Use spherical coordinates to find the volume of the solid inside the torus given by $\rho = 9\sin\phi$.

55. Use spherical coordinates to find the z coordinates of the center of mass of the solid lying between two concentric hemispheres of radii 6 and 8, and having uniform density k.