M252 L8 Practice exam 5 sections 13.1-13.3, 13.5-13.7

1. Evaluate the following integral.

$$\int_9^{4y} \frac{6y}{x}\,dx = \left[6y\ln|x|\right]_{x=9}^{x=4y} = 6y(\ln(4y)-\ln 9) = 6y\ln\left(\frac{4y}{9}\right)$$

(assume $y>0$ otherwise integral diverges)

2. Evaluate the following integral.

$$\int_{4x}^{x^4} \frac{-10y}{x}\,dy = \left[\frac{-5y^2}{x}\right]_{y=4x}^{y=x^4} = -5x^7 - -\frac{5(4x)^2}{x} = 80x-5x^7$$

3. Evaluate the following integral.

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$$\int_y^{\frac{\pi}{11}} \sin^3 3x\cos y\,dx = \cos y\int_y^{\frac{\pi}{11}}(1-\cos^2 3x)\sin 3x\,dx$$

$u = \cos 3x$

$\frac{du}{-3} = \sin(3x)dx$

$$\cos y\left[u-\frac{u^3}{3}\right]_{x=y}^{x=\frac{\pi}{11}}\left(\frac{-1}{3}\right) = \frac{1}{3}\cos y\left(\cos 3y - \frac{\cos^3(3y)}{3} - \cos\left(\frac{3\pi}{11}\right) + \frac{\cos^3\left(\frac{3\pi}{11}\right)}{3}\right)$$

4. Evaluate the following iterated integral.

$$\int_2^6\int_1^6 (4x+y)\,dy\,dx$$

$$\int_2^6\left[4xy+\frac{y^2}{2}\right]_{y=1}^{y=6} dx = \int_2^6 24x+\frac{6^2}{2}-\left(4x+\frac{1}{2}\right)dx$$

$$= \int_2^6 20x+\frac{35}{2}\,dx = \left[10x^2+\frac{35x}{2}\right]_2^6 = 360+105-40-35$$

$$= 390$$

5. Evaluate the following iterated integral.

$$\int_6^7 \int_1^{\sqrt{x}} 2ye^{-x}\, dy\, dx = \int_6^7 \left[y^2 e^{-x}\right]_{y=1}^{y=\sqrt{x}} dx = \int_6^7 xe^{-x} - e^{-x} dx =$$

$$\left[-xe^{-x}\right]_6^7 = \frac{6}{e^6} - \frac{7}{e^7}$$

6. Evaluate the following iterated integral.

$$\int_3^5 \int_y^{-2y} \left(-2+6x^2+6y^2\right) dx\, dy = \int_3^5 \left[-2x+2x^3+6xy^2\right]_{x=y}^{x=-2y} dy$$

$$\int_3^5 (4y-16y^3-12y^3)-(-2y+2y^3+6y^3)dy = \int_3^5 -36y^3+6y\, dy = \left[-9y^4+3y^2\right]_3^5$$

$$-9\cdot625+75+9\cdot81-27 = -5625+75+729-27 = -4848$$

7. Evaluate the following iterated integral.

$$\int_0^{\frac{\pi}{9}} \int_0^{11\cos\theta} r\, dr\, d\theta = \int_0^{\frac{\pi}{9}} \left[\frac{r^2}{2}\right]_0^{11\cos\theta} d\theta = \int_0^{\pi/9} \frac{11^2\cos^2\theta}{2}\, d\theta$$

$$= \frac{121}{4}\left[\theta+\sin\theta\cos\theta\right]_0^{\pi/9} = \frac{121\pi}{36} + \frac{121}{8}\sin\left(\frac{2\pi}{9}\right)$$

8. Evaluate the following improper integral.

$$\int_2^{\infty} \int_0^{7/x} y\, dy\, dx = \int_2^{\infty} \left[\frac{y^2}{2}\right]_0^{7/x} dx = \int_2^{\infty} \frac{49}{2x^2}\, dx$$

$$= \lim_{b\to\infty} \int_2^b \frac{49}{2x^2}\, dx = \lim_{b\to\infty} \left[\frac{-49}{2x}\right]_2^b = \frac{49}{4}$$

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9. Use an iterated integral to find the area of the region bounded by

$\sqrt{x}+\sqrt{y}=2,\ x=0,\ y=0.$

$$\int_{x=0}^{x=4}\int_{y=0}^{y=(2-\sqrt{x})^2} dy\,dx = \int_0^4 (2-\sqrt{x})^2\,dx$$

$$= \int_0^4 4-2\sqrt{x}+x\,dx = \left[4x-\frac{4x^{3/2}}{3}+\frac{x^2}{2}\right]_0^4 = 16-\frac{32}{3}+8=\frac{40}{3}$$

10. Use an iterated integral to find the area of the region bounded by

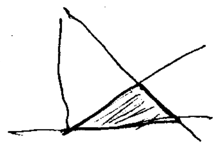
$4x-3y=0,\ x+y=2,\ y=0.$

intersect

$4x-3(2-x)=0$

$7x-6=0$

$x=\frac{6}{7},\ y=\frac{8}{7}$

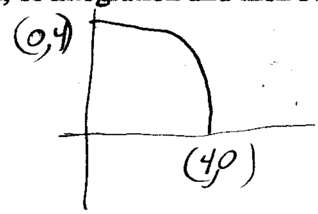


$$\int_{y=0}^{y=\frac{8}{7}}\int_{x=\frac{3}{4}y}^{x=2-y} dx\,dy = \int_0^{8/7} 2-\frac{7}{4}y\,dy$$

$$\left[2y-\frac{7y^2}{8}\right]_0^{8/7} = \frac{16}{7}-\frac{8}{7}=\frac{8}{7}$$

11. Sketch the region, R, of integration and then switch the order of integration for the following integral.

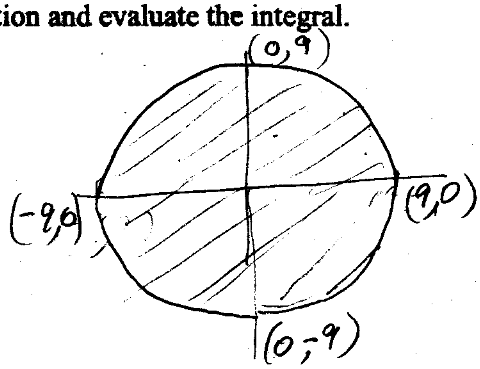
$$\int_0^4\int_0^{\sqrt{16-x^2}} f(x,y)\,dy\,dx$$



$$\int_0^4\int_0^{\sqrt{16-y^2}} f(x,y)\,dx\,dy$$

12. Sketch the region, R, whose area is given by the iterated integral below. Then switch the order of integration and evaluate the integral.

$$\int_{-9}^{9}\int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} dy\,dx$$



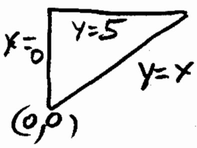
$$\int_{-9}^{9}\int_{-\sqrt{81-y^2}}^{\sqrt{81-y^2}} dx\,dy$$

$$\int_{-9}^{9} 2\sqrt{81-y^2}\,dy = \left[y\sqrt{81-y^2}+81\arcsin\frac{y}{9}\right]_{-9}^{9} = \frac{81\pi}{2}-\frac{-81\pi}{2}=81\pi$$

(integration formula 37 pg A29)

13. **Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.**

$$\int_0^5\int_x^5 e^{-8y^2}\,dy\,dx$$



$$\int_0^5\int_0^y e^{-8y^2}\,dx\,dy = \int_0^5 y e^{-8y^2}\,dy$$

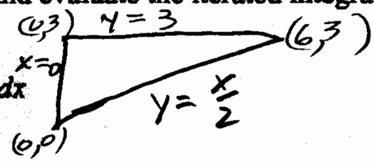
$u = -8y^2$

$du = -16y\,dy$

$$-\frac{1}{16}\int_{y=0}^{y=5} e^{u}\,du = -\frac{1}{16}\left[e^{-8y^2}\right]_0^5 = \frac{1}{16} - e^{-200}$$

14. **Sketch the region R and evaluate the iterated integral.**

$$I = \int_0^6\int_{x/2}^3 (x+y)\,dy\,dx$$



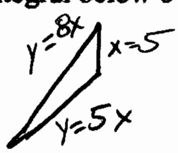
$$\int_0^6\left[xy+\frac{y^2}{2}\right]_{y=\frac{x}{2}}^{y=3} dx$$

$$\int_0^6 3x+\frac{9}{2}-\frac{x^2}{2}-\frac{x^2}{8}\,dx = \left[\frac{-5x^3}{24}+\frac{3x^2}{2}+\frac{9x}{2}\right]_0^6 = \frac{-5\cdot 36}{4}+3\cdot 18+27$$

$$= 54+27-45 = 36$$

15. **Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral below over the region R.**

$$\iint_R \frac{y}{x^2+y^2}\,dA$$



$$\int_0^{40}\int_{x=\frac{y}{8}}^{x=\min\left(\frac{y}{5},5\right)} \frac{y}{x^2+y^2}\,dx\,dy$$

better is

$$\int_0^5\int_{y=5x}^{y=8x} \frac{y}{x^2+y^2}\,dy\,dx$$

R: triangle bounded by $y = 5x$, $y = 8x$, and $x = 5$

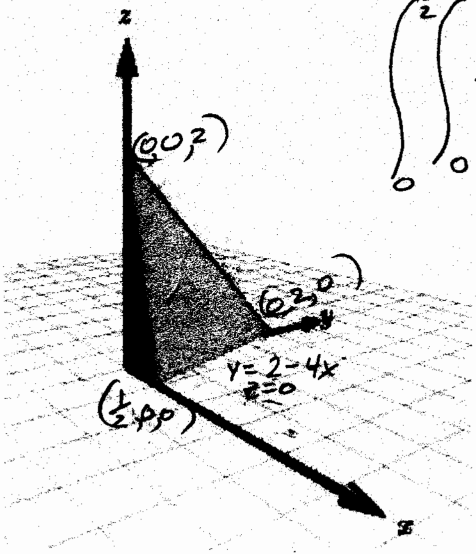
let $u = x^2+y^2$

$du = 2y\,dy$

$$\int_0^5\int_{u=26x^2}^{u=65x^2}\frac{du}{2u} = \int_0^5 \frac{1}{2}\left[\ln u\right]_{26x^2}^{65x^2} dx = \int_0^5 \frac{1}{2}\ln\frac{65}{26}\,dx = \frac{5}{2}\ln\frac{5}{2}$$

16. OMIT

17. Use a double integral to find the volume of the indicated solid.



$4x+y+z=2,\ \ x>0, y>0, z>0$

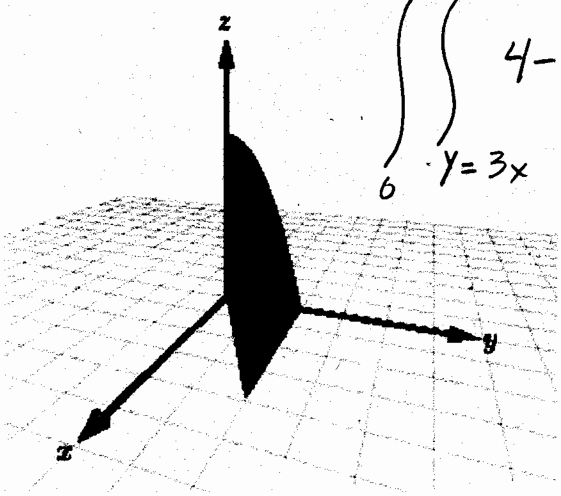
$$\int_0^{\frac{1}{2}}\int_0^{2-4x} z\,dy\,dx = \int_0^{\frac{1}{2}}\int_{y=0}^{y=2-4x} 2-4x-y\,dy\,dx = \int_0^{\frac{1}{2}}\left[2y-4xy-\frac{y^2}{2}\right]_{y=0}^{y=2-4x} dx$$

$$\int_0^{\frac{1}{2}} 2(2-4x)+4x(2-4x)-\frac{(2-4x)^2}{2}\,dx$$

$$\int_0^{\frac{1}{2}}\left(-24x^2+8x+2\right)dx = \left[-8x^3+4x^2+2x\right]_0^{\frac{1}{2}}$$

$$= 1$$

18. Use a double integral to find the volume of the indicated solid.



$z=4-y^2,\ \ z>0,\ \ x>0,\ \ 3x<y<2$

$$\int_0^{\frac{2}{3}}\int_{y=3x}^{2} 4-y^2\,dy\,dx = \int_0^{\frac{2}{3}}\left[4y-\frac{y^3}{3}\right]_{y=3x}^{2} dx$$

$$= \int_0^{\frac{2}{3}}\left(8-\frac{8}{3}-12x+9x^3\right)dx$$

$$\left[\frac{16x}{3}-6x^2+\frac{9x^4}{4}\right]_0^{\frac{2}{3}}$$

$$= \frac{32}{9}-\frac{24}{9}+\frac{4}{9} = \frac{12}{9} = \frac{4}{3}$$

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19. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$z = xy$, $z > 0$, $x > 0$, $7x < y < 4$

$$\int_0^{4/7}\int_{y=7x}^{y=4} xy\,dy\,dx = \int_0^{4/7}\left[\frac{xy^2}{2}\right]_{y=7x}^{y=4} dx$$

$$\int_0^{4/7} 8x - \frac{49x^3}{2}\,dx = \left[4x^2 - \frac{49x^4}{8}\right]_0^{4/7} = 4\left(\frac{4}{7}\right)^2 - \frac{49}{8}\left(\frac{4}{7}\right)^4 = \frac{32}{49}$$

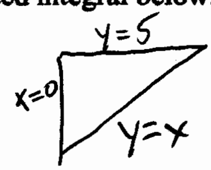
20. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$z = \dfrac{1}{25+y^2}$, $x = 0$, $x = 2$, $y \geq 0$

$$\int_0^2\int_0^{\infty} \frac{1}{25+y^2}\,dy\,dx$$

21. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^5\int_x^5 e^{-0.29y^2}\,dy\,dx$$



$$\int_0^5\int_{x=0}^{x=y} e^{-0.29y^2}\,dx\,dy = \int_0^5 ye^{-.29y^2}\,dy$$

$u = -0.29y^2$

$du = -0.58y\,dy$

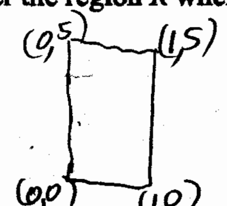
$$-\frac{1}{.58}\int_{y=0}^{y=5} e^u\,du = -\frac{1}{.58}\left[e^{-.29y^2}\right]_{y=0}^{y=5} = \frac{1}{.58}\left(1 - e^{-8.41}\right)$$

22. Find the average value of $f(x,y)$ over the region R where:

Average value $= \dfrac{1}{A}\iint_R f(x,y)\,dA$

$f(x,y) = xy$

$A = 5$



R: rectangle with vertices $(0,0)$, $(1,0)$, $(1,5)$, $(0,5)$

$$\frac{1}{5}\int_0^1\int_0^5 xy\,dy\,dx = \int_0^1\left[\frac{xy^2}{2}\right]_0^5 dx\ \frac{1}{5}$$

$$\frac{1}{5}\int_0^1 \frac{25x}{2}\,dx = \left[\frac{25x^2}{5\cdot 4}\right]_0^1 = \frac{5}{4}$$

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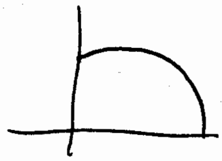
23. **Identify the region of integration for the following integral.**

$$\int_0^{\pi/2}\int_0^{1+\sin\theta} r\theta\,dr\,d\theta$$

First quadrant of the Cardioid



24. **Evaluate the double integral below.**

$$\int_0^{2\pi}\int_0^{6} 5\,r^5 \sin\theta\,dr\,d\theta$$

$$\int_0^{2\pi}\left[\frac{5r^6}{6}\right]_0^6 \sin\theta\,d\theta = 5\cdot 6^5\left[-\cos\theta\right]_0^{2\pi} = 0$$

25. **Evaluate the double integral below.**

$$\int_0^{\pi/7}\int_0^{3} r^3\sin\theta\cos\theta\,dr\,d\theta = \left[\frac{r^4}{4}\right]_0^3 \int_0^{\pi/7}\sin\theta\cos\theta\,d\theta$$

let $u=\sin\theta$
$du=\cos\theta\,d\theta$

$$\left[\frac{r^4}{4}\right]_0^3\left[\frac{u^2}{2}\right]_{\theta=0}^{\theta=\frac{\pi}{7}} = \frac{81}{8}\sin^2\frac{\pi}{7} = 1.9$$

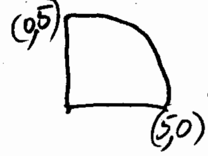
26. **Evaluate the double integral below.**

$$\int_0^{\pi/2}\int_0^{5+3\sin\theta} \theta r\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\left[\frac{\theta r^2}{2}\right]_0^{5+3\sin\theta} d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\theta(5+3\sin\theta)^2\,d\theta = 34.32$$

27. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^5\int_0^{\sqrt{25-x^2}} y\,dy\,dx$$



$$\int_0^5\int_0^{\frac{\pi}{2}} r^2\sin\theta\,d\theta\,dr = \int_0^5 r^2\left[-\cos\theta\right]_0^{\frac{\pi}{2}}dr$$

$$\int_0^5 r^2\,dr = \left[\frac{r^3}{3}\right]_0^5 = \frac{125}{3}$$

28. OMIT

29. OMIT

30. Use a double integral in polar coodinates to find the volume of the solid in the first octant bounded by the graphs of the equations given below.

$$z = x^4 y, \quad x^2 + y^2 = 16$$

$$\int_0^4\int_0^{\frac{\pi}{2}} (r\cos\theta)^4(r\sin\theta)\,r\,d\theta\,dr$$

$$\int_0^4 r^6\int_0^{\frac{\pi}{2}} \cos^4\theta\sin\theta\,d\theta\,dr = \left[-\frac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{2}}\int_0^4 r^6\,dr = \frac{1}{5}\left[\frac{r^7}{7}\right]_0^4$$

$$= \frac{4^7}{5\cdot 7^7}$$

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31. **Use a double integral in polar coodinates to find the volume of the solid inside the hemisphere**

$$z=\sqrt{81-x^2-y^2}$$

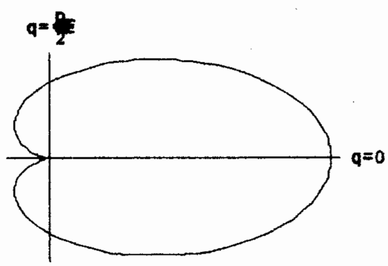
but outside the cylinder

$$x^2+y^2=49.$$

$$\int_0^{2\pi}\int_7^9 \sqrt{81-r^2}\; r\,dr\,d\theta \qquad \text{let } u=81-r^2, \quad du=-2r\,dr$$

$$2\pi\int_{r=7}^{r=9}\frac{\sqrt{u}}{-2}\,du = 2\pi\left[-\frac{u^{3/2}}{3}\right]_{r=7}^{r=9} = \frac{2\pi}{3}\,32^{3/2} \approx 379.13$$

32. **Use a double integral to find the area enclosed by the graph of** $r=5+5\cos\theta$.



$$\int_0^{2\pi}\int_0^{5+5\cos\theta} r\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{5+5\cos\theta} d\theta$$

$$\int_0^{2\pi}\frac{25}{2}(1+\cos\theta)^2\,d\theta \approx 117.8$$

33.

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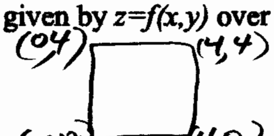
34. **Find the area of the surface given by** $z=f(x,y)$ **over the region** R.

$$f(x,y)=-6-4x+2y$$



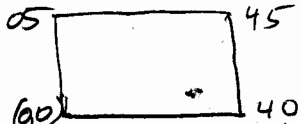
$$f_x=-4 \quad f_y=2$$

R: square with vertices (0,0), (4,0), (4,4), (0,4)

$$\int_0^4\int_0^4\sqrt{1+f_x^2+f_y^2}\,dx\,dy = \int_0^4\int_0^4\sqrt{1+16+4} = \sqrt{21}\cdot 16 \approx 73.32$$

35. Find the area of the surface given by $z = f(x,y)$ over the region R.

$f(x,y) = 10 + 2x - 4y$



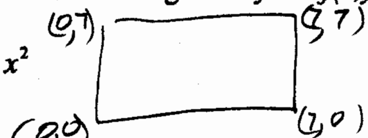
$f_x = 2 \quad f_y = -4$

R: rectangle with vertices (0,0), (4, 0), (4, 5), (0, 5)

$$\int_0^4 \int_0^5 \sqrt{1 + f_x^2 + f_y^2}\, dy\, dx = \int_0^4 \int_0^5 \sqrt{1+16+4} = \sqrt{21} \cdot 20 \approx 91.65$$

36. Find the area of the surface given by $z = f(x,y)$ over the region R.

$f(x,y) = -1 - x^2$



$f_x = -2x \quad f_y = 0$

R: rectangle with vertices (0,0), (7, 0), (7, 7), (0, 7)

$$\int_0^7 \int_0^7 \sqrt{1+4x^2}\, dx\, dy = 14 \left[\frac{1}{2} \left(x \sqrt{x^2 + \frac{1}{4}} + \frac{1}{4} \ln \left| x + \sqrt{x + \frac{1}{4}} \right| \right) \right]_0^7 \approx 349.7$$

37. Find the area of the surface given by $z = f(x,y)$ over the region R.

$f(x,y) = xy \qquad f_x = y \quad f_y = x$

R: $\{(x,y) : x^2 + y^2 \le 121\}$

$u = 1 + r^2$
$du = 2r\,dr$

$$\iint_R \sqrt{1 + x^2 + y^2}\, dA = \int_0^{2\pi} \int_0^{11} \sqrt{1+r^2}\, r\, dr\, d\theta = \pi \int_{r=0}^{r=11} \sqrt{u}\, du = \pi \left[\frac{2u^{\frac{3}{2}}}{3} \right]_1^{122}$$

$$= 897.689\pi$$

$$\approx 2820.17$$

38. Find the area of the surface of the portion of the plane

$z = 5 - 2x - 8y \qquad f_x = -2 \quad f_y = -8$

in the first octant. R: $\{(x,y) \mid 2x + 8y \le 5 \;\&\; x \ge 0 \;\&\; y \ge 0\}$

$$\int_0^{\frac{5}{8}} \int_{x=0}^{x = \frac{5-8y}{2}} \sqrt{1+4+64}\, dx\, dy = \frac{\sqrt{71}}{2} \int_0^{5/8} (5 - 8y)\, dy = \left[5y - 4y^2\right]_0^{\frac{5}{8}} \frac{\sqrt{71}}{2}$$

$$= \left(\frac{25}{8} - \frac{100}{64} \right) \frac{\sqrt{71}}{2} = \frac{50}{64}\sqrt{71} = \frac{25}{32}\sqrt{71}$$

$$\approx 6.58$$

39. **Set up a double integral that gives the area of the surface of the graph of f over the region R.**

$f(x,y)=x^4-3xy+7y^3$

$f_x = 4x^3-3y \qquad f_y = -3x+21y^2$

$R=\{(x,y): -5\le x\le 5, -2\le y\le 2\}$

$$\int_{-5}^{5}\int_{-2}^{2}\sqrt{1+(4x^3-3y)^2+(-3x+21y^2)^2}\,dy\,dx$$

40. **Set up a double integral that gives the area of the surface of the graph of f over the region R.**

$f(x,y)=e^{7xy}$

$f_x = 7ye^{7xy} \qquad f_y = 7xe^{7xy}$

$R=\{(x,y): 0\le x\le 2, 0\le y\le 4\}$

$$\int_0^2\int_0^4\sqrt{1+49y^2e^{14xy}+49x^2e^{14xy}}\,dy\,dx$$

41. **Evaluate the following iterated integral.**

$$\int_0^6\int_0^4\int_0^5(5x+3y+z)\,dx\,dy\,dz = \int_0^6\int_0^4\left[\frac{5x^2}{2}+3xy+xz\right]_{x=0}^{x=5}dy\,dz = \int_0^6\int_0^4\left(\frac{125}{2}+15y+5z\right)dy\,dz$$

$$= \int_0^6\left[\frac{125y}{2}+\frac{15y^2}{2}+5yz\right]_{y=0}^{y=4}dz = \int_0^6 250+120+20z\,dz = \left[370z+10z^2\right]_0^6 = 2580$$

42. **Evaluate the following iterated integral.**

$$\int_1^8\int_0^1\int_0^x 2ze^{-x^2}\,dy\,dx\,dz = \int_1^8\int_0^1\left[2zye^{-x^2}\right]_{y=0}^{y=x}dx\,dz = \int_1^8 z\int_0^1 2xe^{-x^2}\,dx\,dz$$

$u=-x^2$
$du=-2x\,dx$

$$\int_1^8 z\int_{x=0}^{x=1}-e^u\,du\,dz = \int_1^8 z\left[-e^u\right]_0^1 dz = \int_1^8 z\left(1-\frac{1}{e}\right)dz$$

$$= \left[\frac{z^2}{2}\left(1-\frac{1}{e}\right)\right]_1^8 = \frac{63\left(1-\frac{1}{e}\right)}{2}$$

43. **Set up a triple integral for the volume of the solid bounded by the coordinate planes and the plane given below.**

$z = 6 - 2x - 4y$

$$\int_{y=0}^{y=\frac{3}{2}} \int_{x=0}^{x=3-2y} \int_{z=0}^{z=6-2x-4y} dz\,dx\,dy$$

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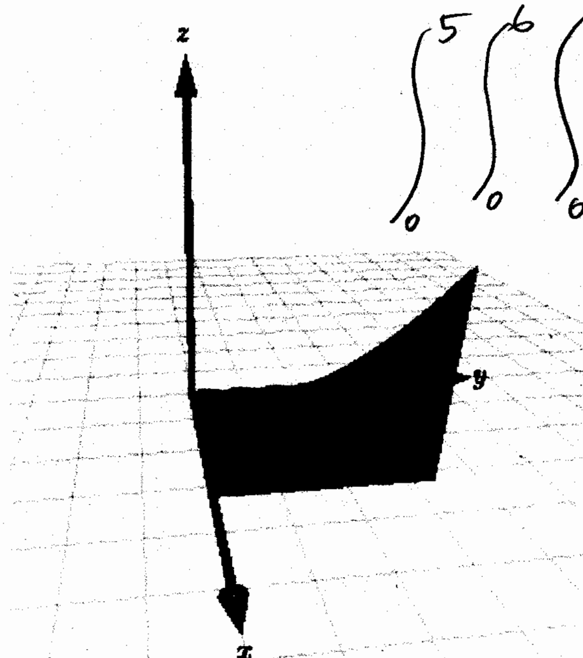
44. **Use a triple integral for the volume of the solid shown below.**



$z = 12xy, \quad 0 \le x \le 5, \quad 0 \le y \le 6$

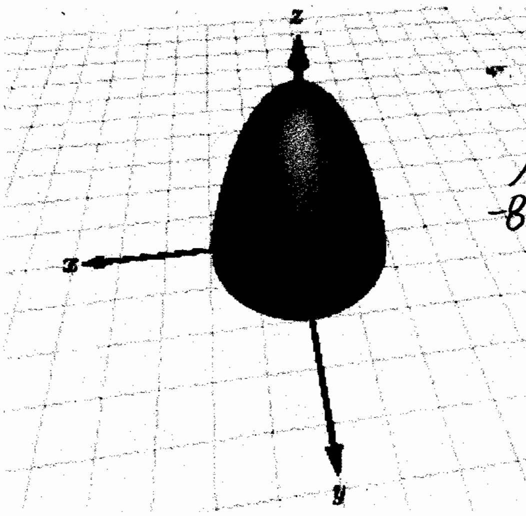
$$\int_0^5 \int_0^6 \int_0^{z=12xy} dz\,dy\,dx$$

$$\int_0^5 \int_0^6 12xy\,dy\,dx$$

$$\int_0^5 \left[6xy^2\right]_0^6 dx$$

$$\left[6^3 \frac{x^2}{2}\right]_0^5 = \frac{6^3 \cdot 25}{2} = 2700$$

45. Use a triple integral for the volume of the solid shown below.



$z = 64 - x^2 - y^2, \ z \geq 0$

$z = 64 - x^2 - y^2$

$$\int_{-8}^{8}\int_{-\sqrt{64-x^2}}^{\sqrt{64-x^2}}\int_{z=0}^{64-x^2-y^2} dz\,dy\,dx$$

$$= \int_0^{2\pi}\int_0^8 (64-r^2)\,r\,dr\,d\theta$$

$$2\pi\left[32r^2 - \frac{r^3}{3}\right]_0^8 = 2\pi\left(32\cdot 64 - \frac{8^3}{3}\right)$$

$$\approx 11795$$

46. OMIT

47. Find the average value of the function f over the region in the first octant bounded by the coordinate planes, and the planes $x = 4$, $y = 3$, and $z = 2$.

$f(x, y, z) = x^3 + y^2 + z^4$

$V = 24$

$$\frac{1}{V}\int_0^4\int_0^3\int_0^2 (x^3 + y^2 + z^4)\,dz\,dy\,dx$$

$$\frac{1}{24}\int_0^4\int_0^3 \left[x^3 z + y^2 z + \frac{z^5}{5}\right]_{z=0}^{z=2} dy\,dx = \frac{1}{24}\int_0^4\int_0^3 \left(2x^3 + 2y^2 + \frac{32}{5}\right) dy\,dx$$

$$\frac{1}{24}\int_0^4 \left[2x^3 y + \frac{2y^3}{3} + \frac{32y}{5}\right]_{y=0}^{y=3} dx \to \frac{1}{24}\int_0^4 \left(6x^3 + 18 + \frac{96}{5}\right) dx = \frac{1}{24}\left[\frac{6x^4}{4} + \frac{186x}{5}\right]_0^4$$

$$= \frac{1}{24}\left(6\cdot 64 + \frac{186\cdot 4}{5}\right) = 22.2$$

48. Evaluate the following iterated integral.

$$\int_0^{\pi/2}\int_0^{1\cos^4\theta}\int_0^{1-r^2} r\sin\theta\, dz\, dr\, d\theta = \int_0^{\frac{\pi}{2}}\int_0^{\cos^4\theta}(1-r^2)\, r\sin\theta\, dr\, d\theta$$

$u=\cos\theta$
$du=-\sin\theta\, d\theta$

$$= \int_0^{\frac{\pi}{2}} \sin\theta\left[\frac{r^2}{2}-\frac{r^4}{4}\right]_{r=0}^{r=\cos^4\theta} d\theta = -\frac{1}{4}\int_0^{\frac{\pi}{2}}(\cos^{16}\theta - 2\cos^8\theta)\sin\theta\, d\theta = -\frac{1}{4}\left[\frac{2u^9}{9}-\frac{u^{17}}{17}\right]_1^0$$

$$= -\frac{1}{4}\left(\frac{1}{17}-\frac{2}{9}\right) = \frac{7}{102}$$

49. Evaluate the following iterated integral.

$$\int_0^{\pi/13}\int_0^{\pi/13}\int_0^{\cos\theta}\rho^2\sin\phi\cos\phi\, d\rho\, d\theta\, d\phi = \int_0^{\frac{\pi}{13}}\int_0^{\frac{\pi}{13}}\left[\frac{\rho^3}{3}\right]_0^{\cos\theta}\sin\phi\cos\phi\, d\theta\, d\phi$$

$$\frac{1}{3}\int_0^{\frac{\pi}{13}}\int_0^{\frac{\pi}{13}}\cos^3\theta\sin\phi\cos\phi\, d\phi\, d\theta = \frac{1}{3}\int_0^{\frac{\pi}{13}}\left[\frac{\cos^2\phi}{2}\right]_0^{\frac{\pi}{13}}\cos^3\theta\, d\theta = \left(\frac{\cos^2\frac{\pi}{13}-1}{6}\right)\int_0^{\frac{\pi}{13}}\cos^3\theta\, d\theta$$

$= -.00224$

50. Evaluate the following iterated integral.

$$\int_0^{2\pi}\int_0^{\pi}\int_3^5\rho^2\sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi}\sin\phi\left[\frac{\rho^3}{3}\right]_3^5 d\phi\, d\theta = \left(\frac{125}{3}-9\right)\int_0^{2\pi}\left[-\cos\phi\right]_0^{\pi} d\theta$$

$$2\left(\frac{125}{3}-9\right)2\pi = \frac{392}{3}\pi$$

51. Convert the integral below from rectangular coordinates to ~~both~~ cylindrical ~~and spherical~~ coordinates, ~~and evaluate the simpler iterated integral.~~

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{x^2+y^2}^{9} x\, dz\, dy\, dx$$

cylindrical

$$\int_0^{2\pi}\int_0^{3}\int_{r^2}^{9} r^2\cos\theta\, dz\, dr\, d\theta$$

52. Convert the integral below from rectangular coordinates to ~~both cylindrical and~~ spherical coordinates, ~~and evaluate the simpler iterated integral.~~

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^5 \rho^3 \sin\phi \, d\rho\, d\theta\, d\phi$$

53. Use cylindrical coordinates to find the volume inside the sphere

$x^2+y^2+z^2=4$

$$\int_0^{\sqrt{2}} \int_0^{2\pi} \int_r^{\sqrt{4-r^2}} r \, dz\, d\theta\, dr$$

and above the upper nappe of the cone $z^2=(x^2+y^2)$.

$$\int_0^{2\pi} \int_0^{\sqrt{2}} r\left(\sqrt{4-r^2}-r\right) dr\, d\theta = 2\pi\left[-\frac{1}{2}\int_{r=0}^{r=\sqrt{2}} \sqrt{u}\, du - \left[\frac{r^3}{3}\right]_0^{\sqrt{2}}\right) = 2\pi\left(\left[\frac{u^{3/2}}{-2}\right]_4^2 - \frac{2\sqrt{2}}{3}\right)$$

$u = 4-r^2$

$du = -2r\,dr$

etc