M252 Final Exam Practice for Chapters 10-13
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1. Find the magnitude of the vector **v** given its initial and terminal points.

   Initial point: $(4,-6,-3)$

   Terminal point: $(7,-1,-1)$

2. Find the unit vector in the direction of **u**.

   $\mathbf{u}=\langle 4,-3,-5\rangle$

   The possible solutions are given to two decimal places.

3. Find (a) $\mathbf{u}\cdot\mathbf{v}$ (b) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v}$ (c) $\mathbf{u}\cdot(4\mathbf{v})$ given the vectors **u** and **v.**

   $\mathbf{u}=\langle 1,6\rangle, \quad \mathbf{v}=\langle -3,-1\rangle$

   (a) $\mathbf{u}\cdot\mathbf{v}$ (b) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v}$ (c) $\mathbf{u}\cdot(4\mathbf{v})$

4. Find the angle between the vectors for **u** and **v** given below.

   $\mathbf{u}=\langle 2,6\rangle, \quad \mathbf{v}=\langle -2,3\rangle$

5. Determine whether **u** and **v** are orthogonal, parallel, or neither.

$$\mathbf{u} = \langle 4, -5 \rangle, \quad \mathbf{v} = \langle 4, -5 \rangle$$

6. Find the direction cosines of the vector **u** given below.

$$\mathbf{u} = \langle 2, 5, -2 \rangle$$

7. Find the projection of **u** onto **v,** and the vector component of **u** orthogonal to **v**.

$$\mathbf{u} = \langle 1, 9 \rangle, \quad \mathbf{v} = \langle 0, 3 \rangle$$

Projection of **u** onto **v**

Component of **u** orthogonal to **v**

8. Given the vectors **u and v,** find $\mathbf{u} \times \mathbf{v}$ and $\mathbf{v} \times \mathbf{v}$.

$$\mathbf{u} = \langle 1, 9, 2 \rangle, \quad \mathbf{v} = \langle -1, 2, 3 \rangle$$

$\mathbf{u} \times \mathbf{v}$

$\mathbf{v} \times \mathbf{v}$

9. Find a set of parametric equations of the line through the point $(-5, 5, 2)$ parallel to the vector **v**=(7, 5,4).

10. Find an equation of a plane passing through the point given and perpendicular to the given vector.

    Point: $(6,8,7)$ Vector $\mathbf{v} = \langle 7,8,2 \rangle$

11. Find the distance between the point $(1, -2, -1)$ and the plane given below.

    $-5x - 10y + 5z = 15$

12. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

    $9x^2 + 9y^2 = 2x$

13. Find an equation in rectangular coordinates for the equation given in spherical coordinates.

    $\rho = 4\csc\varphi \; sec\theta$

14. Convert the following point from cylindrical coordinates to spherical coordinates.

    $\left(8, \frac{\pi}{6}, 8\right)$

15. Represent the following curve by a vector valued function.

$$\frac{x^2}{49}+\frac{y^2}{16}=1,\ x>0$$

16. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$z=\frac{x^2}{81}+\frac{y^2}{4},\ y+11x=0$	$x=t$

17. The position vector $\mathbf{r}$ describes the path of an object moving in the *xy*-plane. Find the velocity and acceleration vectors at the given point.

$$\mathbf{r}(t)=-4t^4\mathbf{i}+5t^4\mathbf{j},\quad (-324,405)$$

18. The position vector $\mathbf{r}$ describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

$$\mathbf{r}(t)=4\,\mathbf{i}+3t^2\,\mathbf{j}-8t^2\,\mathbf{k}$$

Velocity Speed Acceleration

19. Use the given acceleration function to find the velocity and position vector. Then find the position at time t = 3.

$$\mathbf{a}(t)=10\mathbf{i}+8\,\mathbf{j}+8\mathbf{k},\quad \mathbf{v}(0)=3\mathbf{k},\quad \mathbf{r}(0)=\mathbf{0}$$

20. Find the unit tangent vector **T(**t**)** and then use it to find a set of parametric equations for the line tangent to the space curve given below at the given point.

$$\mathbf{r}(t) = -5t\,\mathbf{i} - 4t^2\,\mathbf{j} + 5t\mathbf{k}, \quad t = 5$$

21. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = t\ \mathbf{i} + 6t^2\,\mathbf{j}, \quad t = 1$$

22. Find the length of the plane curve given below.

$$\mathbf{r}(t) = 4\ \mathbf{i} + 3t^2\ \mathbf{j}, \quad [0,6]$$

23. Find the length of the space curve given below.

$$\mathbf{r}(t) = 3\ \mathbf{i} + 4\cos t\ \mathbf{j} + 4\ \sin t\ \mathbf{k}, \qquad [0,3]$$

24. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = x^2 + 9xy + 7y^2$$

25. Find the total differential of the function $z = \dfrac{x^8}{y}$.

26. Let $w = xy$, where $x = 2\sin t$ and $y = -7\cos t$. Find $\dfrac{dw}{dt}$.

27. Let $w = x^4 + y^4$, where $x = 3s + t$, $y = 3s - t$. Find $\dfrac{\partial w}{ds}$ and $\dfrac{\partial w}{dt}$ and evaluate each partial derivative at the point $s = -2$, $t = 2$.

28. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x, y) = 10x - 10xy + 9y, \quad P(1,6), \quad \vec{\mathbf{v}} = \frac{1}{2}\left(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}\right)$$

29. Find the gradient of the function at the given point.

$$f(x, y) = 6x - 8y^2 + 7, \quad (6,1)$$

30. Find a unit normal vector to the surface $x + y + z = 6$ at the point (3,0,3).

31. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 11$ at the point (3,1,1).

32. Find an equation of the tangent plane to the surface $g(x, y) = x^2 - y^2$ at the point $(2, 7, -45)$.

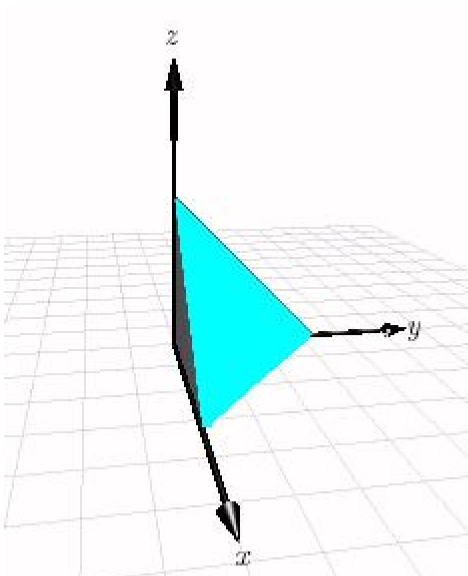
33. Find an equation of the tangent plane to the surface $x^2 + 5y^2 + z^2 = 160$ at the point $(4, -4, 8)$.

34. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xyz = 8$ at the point $(1, 4, 2)$.

35. Sketch the region R and evaluate the iterated integral.

$$I = \int_0^3 \int_0^{2x} (x + y)\,dy\,dx$$

36. Use a double integral to find the volume of the indicated solid.



$4x + 4y + 4z = 5, \;\; x > 0, y > 0, z > 0$

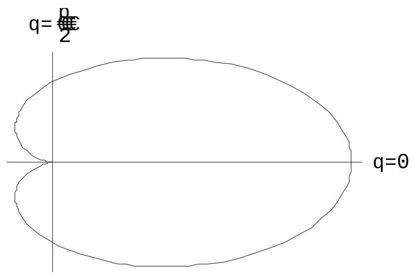
37. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$z = xy^3, \;\; z > 0, \;\; x > 0, \;\; 3x < y < 4$

38. Use a double integral in polar coodinates to find the volume of the solid in the first octant bounded by the graphs of the equations given below.

$z = x^3 y, \quad x^2 + y^2 = 25$

39. Use a double integral to find the area enclosed by the graph of $r = 3 + 3\cos\theta$.



40. Find the area of the surface given by $z=f(x,y)$ over the region R.

$f(x, y) = -6 - 5x + 4y$

R: square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$

41. Find the area of the surface given by $z = f(x,y)$ over the region R.

$f(x, y) = 1 - x^2$

R: rectangle with vertices $(0,0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$

42. Find the area of the surface given by $z = f(x,y)$ over the region R.

$f(x, y) = xy$

R: $\{(x, y) : x^2 + y^2 \le 9\}$

43. Evaluate the following iterated integral.

$$\int_0^2 \int_0^1 \int_0^1 (-4x + 5y + z)\, dx\, dy\, dz$$

44. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$$