

1. Find the magnitude of the vector \mathbf{v} given its initial and terminal points.

Initial point: $(4, -6, -3)$

Terminal point: $(7, -1, -1)$

2. Find the unit vector in the direction of \mathbf{u} .

$$\mathbf{u} = \langle 4, -3, -5 \rangle$$

The possible solutions are given to two decimal places.

3. Find (a) $\mathbf{u} \cdot \mathbf{v}$ (b) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$ (c) $\mathbf{u} \cdot (4\mathbf{v})$ given the vectors \mathbf{u} and \mathbf{v} .

$$\mathbf{u} = \langle 1, 6 \rangle, \quad \mathbf{v} = \langle -3, -1 \rangle$$

(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$

(c) $\mathbf{u} \cdot (4\mathbf{v})$

4. Find the angle between the vectors for \mathbf{u} and \mathbf{v} given below.

$$\mathbf{u} = \langle 2, 6 \rangle, \quad \mathbf{v} = \langle -2, 3 \rangle$$

5. Determine whether \mathbf{u} and \mathbf{v} are orthogonal, parallel, or neither.

$$\mathbf{u} = \langle 4, -5 \rangle, \quad \mathbf{v} = \langle 4, -5 \rangle$$

6. Find the direction cosines of the vector \mathbf{u} given below.

$$\mathbf{u} = \langle 2, 5, -2 \rangle$$

7. Find the projection of \mathbf{u} onto \mathbf{v} , and the vector component of \mathbf{u} orthogonal to \mathbf{v} .

$$\mathbf{u} = \langle 1, 9 \rangle, \quad \mathbf{v} = \langle 0, 3 \rangle$$

Projection of \mathbf{u} onto \mathbf{v}

Component of \mathbf{u} orthogonal to \mathbf{v}

8. Given the vectors \mathbf{u} and \mathbf{v} , find $\mathbf{u} \times \mathbf{v}$ and $\mathbf{v} \times \mathbf{v}$.

$$\mathbf{u} = \langle 1, 9, 2 \rangle, \quad \mathbf{v} = \langle -1, 2, 3 \rangle$$

$\mathbf{u} \times \mathbf{v}$

$\mathbf{v} \times \mathbf{v}$

9. Find a set of parametric equations of the line through the point $(-5, 5, 2)$ parallel to the vector $\mathbf{v} = (7, 5, 4)$.

10. Find an equation of a plane passing through the point given and perpendicular to the given vector.

Point: $(6, 8, 7)$ Vector $\mathbf{v} = \langle 7, 8, 2 \rangle$

11. Find the distance between the point $(1, -2, -1)$ and the plane given below.

$$-5x - 10y + 5z = 15$$

12. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$$9x^2 + 9y^2 = 2x$$

13. Find an equation in rectangular coordinates for the equation given in spherical coordinates.

$$\rho = 4 \csc \varphi \sec \theta$$

14. Convert the following point from cylindrical coordinates to spherical coordinates.

$$\left(8, \frac{\pi}{6}, 8 \right)$$

15. Represent the following curve by a vector valued function.

$$\frac{x^2}{49} + \frac{y^2}{16} = 1, x > 0$$

16. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces

$$z = \frac{x^2}{81} + \frac{y^2}{4}, y + 11x = 0$$

Parameter

$$x = t$$

17. The position vector \mathbf{r} describes the path of an object moving in the xy -plane. Find the velocity and acceleration vectors at the given point.

$$\mathbf{r}(t) = -4t^4\mathbf{i} + 5t^4\mathbf{j}, \quad (-324, 405)$$

18. The position vector \mathbf{r} describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

$$\mathbf{r}(t) = 4\mathbf{i} + 3t^2\mathbf{j} - 8t^2\mathbf{k}$$

Velocity

Speed

Acceleration

19. Use the given acceleration function to find the velocity and position vector. Then find the position at time $t = 3$.

$$\mathbf{a}(t) = 10\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}, \quad \mathbf{v}(0) = 3\mathbf{k}, \quad \mathbf{r}(0) = \mathbf{0}$$

20. Find the unit tangent vector $\mathbf{T}(t)$ and then use it to find a set of parametric equations for the line tangent to the space curve given below at the given point.

$$\mathbf{r}(t) = -5t\mathbf{i} - 4t^2\mathbf{j} + 5t\mathbf{k}, \quad t = 5$$

21. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = t\mathbf{i} + 6t^2\mathbf{j}, \quad t = 1$$

22. Find the length of the plane curve given below.

$$\mathbf{r}(t) = 4\mathbf{i} + 3t^2\mathbf{j}, \quad [0,6]$$

23. Find the length of the space curve given below.

$$\mathbf{r}(t) = 3\mathbf{i} + 4\cos t\mathbf{j} + 4\sin t\mathbf{k}, \quad [0,3]$$

24. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = x^2 + 9xy + 7y^2$$

25. Find the total differential of the function $z = \frac{x^8}{y}$.

26. Let $w = xy$, where $x = 2\sin t$ and $y = -7\cos t$. Find $\frac{dw}{dt}$.

27. Let $w = x^4 + y^4$, where $x = 3s + t$, $y = 3s - t$. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ and evaluate each partial derivative at the point $s = -2$, $t = 2$.

28. Find the directional derivative of the function at P in the direction of \vec{v} .

$$f(x, y) = 10x - 10xy + 9y, \quad P(1, 6), \quad \vec{v} = \frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$$

29. Find the gradient of the function at the given point.

$$f(x, y) = 6x - 8y^2 + 7, \quad (6, 1)$$

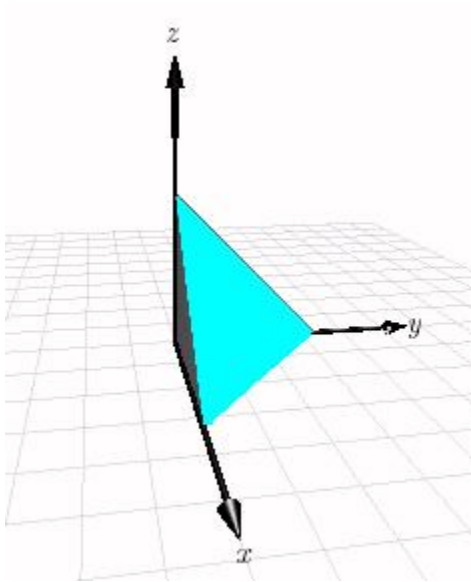
30. Find a unit normal vector to the surface $x + y + z = 6$ at the point $(3, 0, 3)$.

31. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 11$ at the point $(3, 1, 1)$.

32. Find an equation of the tangent plane to the surface $g(x, y) = x^2 - y^2$ at the point $(2, 7, -45)$.
33. Find an equation of the tangent plane to the surface $x^2 + 5y^2 + z^2 = 160$ at the point $(4, -4, 8)$.
34. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xyz = 8$ at the point $(1, 4, 2)$.
35. Sketch the region R and evaluate the iterated integral.

$$I = \int_0^3 \int_0^{2x} (x + y) dy dx$$

36. Use a double integral to find the volume of the indicated solid.



$$4x + 4y + 4z = 5, \quad x > 0, y > 0, z > 0$$

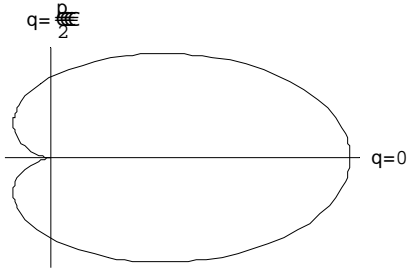
37. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$$z = xy^3, \quad z > 0, \quad x > 0, \quad 3x < y < 4$$

38. Use a double integral in polar coordinates to find the volume of the solid in the first octant bounded by the graphs of the equations given below.

$$z = x^3y, \quad x^2 + y^2 = 25$$

39. Use a double integral to find the area enclosed by the graph of $r = 3 + 3 \cos \theta$.



40. Find the area of the surface given by $z=f(x,y)$ over the region R .

$$f(x, y) = -6 - 5x + 4y$$

R : square with vertices $(0, 0)$, $(2, 0)$, $(2, 2)$, $(0, 2)$

41. Find the area of the surface given by $z = f(x,y)$ over the region R .

$$f(x, y) = 1 - x^2$$

R : rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$

42. Find the area of the surface given by $z = f(x,y)$ over the region R .

$$f(x, y) = xy$$

R : $\{(x, y) : x^2 + y^2 \leq 9\}$

43. Evaluate the following iterated integral.

$$\int_0^2 \int_0^1 \int_0^1 (-4x + 5y + z) \, dx \, dy \, dz$$

44. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx$$