Exercise Set 1.1

Appendix B contains either full or partial solutions to all exercises with blue numbers. When the solution is not complete, the exercise number has an ***H*** next to it. A ∗ next to an exercise number signals that the exercise is more challenging than usual. Be careful not to get into the habit of turning to the solutions too quickly. Make every effort to work exercises on your own before checking your answers. See the Preface for additional sources of assistance and further study.

In each of 1–6, fill in the blanks using a variable or variables to rewrite the given statement.

1. Is there a real number whose square is -1?

a. Is there a real number x such that ____?

b. Does there exist ____ such that $x^2 = -1$?

2. Is there an integer that has a remainder of 2 when it is divided by 5 and a remainder of 3 when it is divided by 6?

a. Is there an integer n such that n has ____?

b. Does there exist ____ such that if n is divided by 5 the remainder is 2 and if ____?

Note: There are integers with this property. Can you think of one?

3. Given any two real numbers, there is a real number in between.

a. Given any two real numbers a and b, there is a real number c such that c is ____.

b. For any two ____, ____ such that $a < c < b$.

4. Given any real number, there is a real number that is greater.

a. Given any real number r, there is ____ s such that s is ____.

b. For any ____, ____ such that $s > r$.

5. The reciprocal of any positive real number is positive.

a. Given any positive real number r, the reciprocal of ____.

b. For any real number r, if r is ____, then ____.

c. If a real number r ____, then ____.

6. The cube root of any negative real number is negative.

a. Given any negative real number s, the cube root of ____.

b. For any real number s, if s is ____, then ____.

c. If a real number s ____, then ____.

7. Rewrite the following statements less formally, without using variables. Determine, as best as you can, whether the statements are true or false.

a. There are real numbers u and v with the property that $u + v < u - v$.

b. There is a real number x such that $x^2 < x$.

c. For all positive integers n, $n^2 \geq n$.

d. For all real numbers a and b, $|a + b| \leq |a| + |b|$.

In each of 8–13, fill in the blanks to rewrite the given statement.

8. For all objects J, if J is a square then J has four sides.

a. All squares ____.

b. Every square ____.

c. If an object is a square, then it ____.

d. If J ____, then J ____.

e. For all squares J, ____.

9. For all equations E, if E is quadratic then E has at most two real solutions.

a. All quadratic equations ____.

b. Every quadratic equation ____.

c. If an equation is quadratic, then it ____.

d. If E ____, then E ____.

e. For all quadratic equations E, ____.

10. Every nonzero real number has a reciprocal.

a. All nonzero real numbers ____.

b. For all nonzero real numbers r, there is ____ for r.

c. For all nonzero real numbers r, there is a real number s such that ____.

11. Every positive number has a positive square root.

a. All positive numbers ____.

b. For any positive number e, there is ____ for e.

c. For all positive numbers e, there is a positive number r such that ____.

12. There is a real number whose product with every number leaves the number unchanged.

a. Some ____ has the property that its ____.

b. There is a real number r such that the product of r ____.

c. There is a real number r with the property that for every real number s, ____.

13. There is a real number whose product with every real number equals zero.

a. Some ____ has the property that its ____.

b. There is a real number a such that the product of a ____.

c. There is a real number a with the property that for every real number b, ____.