Exercise Set 1.3

1. Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: For all $(x, y) \in A \times B$,

 $(x, y) \in R$ means that $\frac{y}{x}$ is an integer.

- a. Is 4 R 6? Is 4 R 8? Is $(3, 8) \in R$? Is $(2, 10) \in R$?
- b. Write R as a set of ordered pairs.
- c. Write the domain and co-domain of R.
- d. Draw an arrow diagram for R.
- 2. Let $C = D = \{-3, -2, -1, 1, 2, 3\}$ and define a relation S from C to D as follows: For all $(x, y) \in C \times D$,

$$(x, y) \in S$$
 means that $\frac{1}{x} - \frac{1}{y}$ is an integer.

- a. Is 2 S 2? Is -1S 1? Is $(3, 3) \in S$? Is $(3, -3) \in S$?
- b. Write S as a set of ordered pairs.
- c. Write the domain and co-domain of S.
- d. Draw an arrow diagram for S.
- **3.** Let $E = \{1, 2, 3\}$ and $F = \{-2, -1, 0\}$ and define a relation *T* from *E* to *F* as follows: For all $(x, y) \in E \times F$,

$$(x, y) \in T$$
 means that $\frac{x-y}{3}$ is an integer.

- a. Is 3T 0? Is 1T(-1)? Is $(2, -1) \in T$? Is $(3, -2) \in T$?
- b. Write T as a set of ordered pairs.
- c. Write the domain and co-domain of T.
- d. Draw an arrow diagram for T.
- 4. Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows: For all $(x, y) \in G \times H$,

$$(x, y) \in V$$
 means that $\frac{x - y}{4}$ is an integer.

a. Is
$$2 V 6$$
? Is $(-2)V (-6)$? Is $(0, 6) \in V$? Is $(2, 4) \in V$?

- b. Write V as a set of ordered pairs.
- c. Write the domain and co-domain of V.
- d. Draw an arrow diagram for V.
- 5. Define a relation S from **R** to **R** as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$,

 $(x, y) \in S$ means that $x \ge y$.

- a. Is (2, 1) ∈ S? Is (2, 2) ∈ S? Is 2 S 3? Is (-1) S (-2)?
 b. Draw the graph of S in the Cartesian plane.
- 6. Define a relation *R* from **R** to **R** as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$,

$$(x, y) \in R$$
 means that $y = x^2$.

- a. Is (2, 4) ∈ R? Is (4, 2) ∈ R? Is (-3) R 9? Is 9 R (-3)?
 b. Draw the graph of R in the Cartesian plane.
- 7. Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define relations R, S, and T from A to B as follows: For all $(x, y) \in A \times B$, $(x, y) \in R$ means that $x \ge y$. $(x, y) \in S$ means that $\frac{x - y}{2}$ is an integer.
 - $T = \{(4, 7), (6, 5), (6, 7)\}.$
 - a. Draw arrow diagrams for R, S, and T.
 - b. Indicate whether any of the relations R, S, and T are functions.
- 8. Let $A = \{2, 4\}$ and $B = \{1, 3, 5\}$ and define relations U, V, and W from A to B as follows: For all $(x, y) \in A \times B$,

$$(x, y) \in U$$
 means that $y - x > 2$.
 $(x, y) \in V$ means that $y - 1 = \frac{x}{2}$
 $W = \{(2, 5), (4, 1), (2, 3)\}.$

- a. Draw arrow diagrams for U, V, and W.
- b. Indicate whether any of the relations U, V, and W are functions.
- 9. a. Find all relations from $\{0,1\}$ to $\{1\}$.
 - b. Find all functions from $\{0,1\}$ to $\{1\}$.
 - c. What fraction of the relations from $\{0,1\}$ to $\{1\}$ are functions?
- Find four relations from {a, b} to {x, y} that are not functions from {a, b} to {x, y}.
- 11. Define a relation P from \mathbf{R}^+ to \mathbf{R} as follows: For all real numbers x and y with x > 0,

$$(x, y) \in P$$
 means that $x = y^2$.

Is P a function? Explain.

12. Define a relation T from **R** to **R** as follows: For all real numbers x and y,

$$(x, y) \in T$$
 means that $y^2 - x^2 = 1$.

Is T a function? Explain.

13. Let $A = \{-1, 0, 1\}$ and $B = \{t, u, v, w\}$. Define a function $F: A \rightarrow B$ by the following arrow diagram:



- a. Write the domain and co-domain of F.
 b. Find F(-1), F(0), and F(1).
- 14. Let $C = \{1, 2, 3, 4\}$ and $D = \{a, b, c, d\}$. Define a function $G: C \rightarrow D$ by the following arrow diagram:



a. Write the domain and co-domain of G.
b. Find G(1), G(2), G(3), and G(4).

15. Let $X = \{2, 4, 5\}$ and $Y = \{1, 2, 4, 6\}$. Which of the following arrow diagrams determine functions from X to Y?





- 16. Let f be the squaring function defined in Example 1.3.6. Find f(-1), f(0), and $f\left(\frac{1}{2}\right)$.
- 17. Let g be the successor function defined in Example 1.3.6. Find g(-1000), g(0), and g(999).
- 18. Let *h* be the constant function defined in Example 1.3.6. Find $h\left(-\frac{12}{5}\right)$, $h\left(\frac{0}{1}\right)$, and $h\left(\frac{9}{17}\right)$.
- **19.** Define functions f and g from **R** to **R** by the following formulas: For all $x \in \mathbf{R}$,

$$f(x) = 2x$$
 and $g(x) = \frac{2x^3 + 2x}{x^2 + 1}$.

Does f = g? Explain.

20. Define functions H and K from R to R by the following formulas: For all $x \in \mathbf{R}$,

$$H(x) = (x-2)^2$$
 and $K(x) = (x-1)(x-3) + 1$.

Does H = K? Explain.