## Exercise Set 1.3

1. Let $A=\{2,3,4\}$ and $B=\{6,8,10\}$ and define a relation $R$ from $A$ to $B$ as follows: For all $(x, y) \in A \times B$, $(x, y) \in R \quad$ means that $\quad \frac{y}{x}$ is an integer.
a. Is $4 R 6$ ? Is $4 R$ ? Is $(3,8) \in R$ ? Is $(2,10) \in R$ ?
b. Write $R$ as a set of ordered pairs.
c. Write the domain and co-domain of $R$.
d. Draw an arrow diagram for $R$.
2. Let $C=D=\{-3,-2,-1,1,2,3\}$ and define a relation $S$ from $C$ to $D$ as follows: For all $(x, y) \in C \times D$,

$$
(x, y) \in S \quad \text { means that } \frac{1}{x}-\frac{1}{y} \text { is an integer. }
$$

a. Is $2 S 2$ ? Is $-1 S-1$ ? Is $(3,3) \in S$ ? Is $(3,-3) \in S$ ?
b. Write $S$ as a set of ordered pairs.
c. Write the domain and co-domain of $S$.
d. Draw an arrow diagram for $S$.
3. Let $E=\{1,2,3\}$ and $F=\{-2,-1,0\}$ and define a relation $T$ from $E$ to $F$ as follows: For all $(x, y) \in E \times F$,

$$
(x, y) \in T \quad \text { means that } \frac{x-y}{3} \text { is an integer. }
$$

a. Is $3 T 0$ ? Is $1 T(-1)$ ? Is $(2,-1) \in T$ ? Is $(3,-2) \in T$ ?
b. Write $T$ as a set of ordered pairs.
c. Write the domain and co-domain of $T$.
d. Draw an arrow diagram for $T$.
4. Let $G=\{-2,0,2\}$ and $H=\{4,6,8\}$ and define a relation $V$ from $G$ to $H$ as follows: For all $(x, y) \in G \times H$,

$$
(x, y) \in V \quad \text { means that } \frac{x-y}{4} \text { is an integer. }
$$

a. Is $2 V 6$ ? Is $(-2) V(-6) ?$ Is $(0,6) \in V$ ? Is $(2,4) \in V$ ?
b. Write $V$ as a set of ordered pairs.
c. Write the domain and co-domain of $V$.
d. Draw an arrow diagram for $V$.
5. Define a relation $S$ from $\mathbf{R}$ to $\mathbf{R}$ as follows:

For all $(x, y) \in \mathbf{R} \times \mathbf{R}$,

$$
(x, y) \in S \quad \text { means that } \quad x \geq y
$$

a. Is $(2,1) \in S$ ? Is $(2,2) \in S$ ? Is $2 S 3$ ? Is $(-1) S(-2)$ ?
b. Draw the graph of $S$ in the Cartesian plane.
6. Define a relation $R$ from $\mathbf{R}$ to $\mathbf{R}$ as follows:

For all $(x, y) \in \mathbf{R} \times \mathbf{R}$,

$$
(x, y) \in R \quad \text { means that } \quad y=x^{2}
$$

a. Is $(2,4) \in R$ ? Is $(4,2) \in R$ ? Is $(-3) R 9$ ? Is $9 R(-3)$ ?
b. Draw the graph of $R$ in the Cartesian plane.
7. Let $A=\{4,5,6\}$ and $B=\{5,6,7\}$ and define relations $R$, $S$, and $T$ from $A$ to $B$ as follows:
For all $(x, y) \in A \times B$,

$$
\begin{aligned}
& (x, y) \in R \quad \text { means that } \quad x \geq y \\
& (x, y) \in S \quad \text { means that } \quad \frac{x-y}{2} \text { is an integer. } \\
& T=\{(4,7),(6,5),(6,7)\} .
\end{aligned}
$$

a. Draw arrow diagrams for $R, S$, and $T$.
b. Indicate whether any of the relations $R, S$, and $T$ are functions.
8. Let $A=\{2,4\}$ and $B=\{1,3,5\}$ and define relations $U, V$, and $W$ from $A$ to $B$ as follows: For all $(x, y)$ $\in A \times B$,

$$
\begin{aligned}
& (x, y) \in U \quad \text { means that } \quad y-x>2 \\
& (x, y) \in V \quad \text { means that } \quad y-1=\frac{x}{2} \\
& W=\{(2,5),(4,1),(2,3)\} .
\end{aligned}
$$

a. Draw arrow diagrams for $U, V$, and $W$.
b. Indicate whether any of the relations $U, V$, and $W$ are functions.
9. a. Find all relations from $\{0,1\}$ to $\{1\}$.
b. Find all functions from $\{0,1\}$ to $\{1\}$.
c. What fraction of the relations from $\{0,1\}$ to $\{1\}$ are functions?
10. Find four relations from $\{a, b\}$ to $\{x, y\}$ that are not functions from $\{a, b\}$ to $\{x, y\}$.
11. Define a relation $P$ from $\mathbf{R}^{+}$to $\mathbf{R}$ as follows: For all real numbers $x$ and $y$ with $x>0$,

$$
(x, y) \in P \quad \text { means that } \quad x=y^{2}
$$

Is $P$ a function? Explain.
12. Define a relation $T$ from $\mathbf{R}$ to $\mathbf{R}$ as follows: For all real numbers $x$ and $y$,

$$
(x, y) \in T \quad \text { means that } \quad y^{2}-x^{2}=1
$$

Is $T$ a function? Explain.
13. Let $A=\{-1,0,1\}$ and $B=\{t, u, v, w\}$. Define a function $F: A \rightarrow B$ by the following arrow diagram:

a. Write the domain and co-domain of $F$.
b. Find $F(-1), F(0)$, and $F(1)$.
14. Let $C=\{1,2,3,4\}$ and $D=\{a, b, c, d\}$. Define a function $G: C \rightarrow D$ by the following arrow diagram:

a. Write the domain and co-domain of $G$.
b. Find $G(1), G(2), G(3)$, and $G(4)$.
15. Let $X=\{2,4,5\}$ and $Y=\{1,2,4,6\}$. Which of the following arrow diagrams determine functions from $X$ to $Y$ ?
a.

b.

c.

d.

e.

16. Let $f$ be the squaring function defined in Example 1.3.6.

Find $f(-1), f(0)$, and $f\left(\frac{1}{2}\right)$.
17. Let $g$ be the successor function defined in Example 1.3.6.

Find $g(-1000), g(0)$, and $g(999)$.
18. Let $h$ be the constant function defined in Example 1.3.6. Find $h\left(-\frac{12}{5}\right), h\left(\frac{0}{1}\right)$, and $h\left(\frac{9}{17}\right)$.
19. Define functions $f$ and $g$ from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas: For all $x \in \mathbf{R}$,

$$
f(x)=2 x \quad \text { and } \quad g(x)=\frac{2 x^{3}+2 x}{x^{2}+1}
$$

Does $f=g$ ? Explain.
20. Define functions $H$ and $K$ from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas: For all $x \in \mathbf{R}$,

$$
H(x)=(x-2)^{2} \quad \text { and } \quad K(x)=(x-1)(x-3)+1
$$

Does $H=K$ ? Explain.

