Exercise Set 2.1*

In each of 1–4 represent the common form of each argument using letters to stand for component sentences, and fill in the blanks so that the argument in part (b) has the same logical form as the argument in part (a).

1. a. If all integers are rational, then the number 1 is rational.
All integers are rational.
Therefore, the number 1 is rational.

b. If all algebraic expressions can be written in prefix notation, then ______________________.
______________________.
Therefore, $(a+2b)(a^2-b)$ can be written in prefix notation.

2. a. If all computer programs contain errors, then this program contains an error.
This program does not contain an error.
Therefore, it is not the case that all computer programs contain errors.

b. If ______, then ______.
2 is not odd.
Therefore, it is not the case that all prime numbers are odd.

3. a. This number is even or this number is odd.
This number is not even.
Therefore, this number is odd.

b. ______ or logic is confusing.
My mind is not shot.
Therefore, ______.

4. a. If n is divisible by 6, then n is divisible by 3.
If n is divisible by 3, then the sum of the digits of n is divisible by 3.
Therefore, if n is divisible by 6, then the sum of the digits of n is divisible by 3.
(Assume that n is a particular, fixed integer.)

b. If this function is ____ then this function is differentiable.
If this function is ____ then this function is continuous.
Therefore, if this function is a polynomial, then this function ____.

5. Indicate which of the following sentences are statements.
a. 1,024 is the smallest four-digit number that is a perfect square.
b. She is a mathematics major.
c. $128 = 2^6$ d. $x = 2^6$

Write the statements in 6–9 in symbolic form using the symbols ~, ∨, and ∧ and the indicated letters to represent component statements.

6. Let s = "stocks are increasing" and i = "interest rates are steady."
a. Stocks are increasing but interest rates are steady.
b. Neither are stocks increasing nor are interest rates steady.

7. Juan is a math major but not a computer science major. (m = "Juan is a math major," c = "Juan is a computer science major")

8. Let h = "John is healthy," w = "John is wealthy," and s = "John is wise."
a. John is healthy and wealthy but not wise.
b. John is not wealthy but he is healthy and wise.
c. John is neither healthy, wealthy, nor wise.
d. John is neither wealthy nor wise, but he is healthy.
e. John is wealthy, but he is not both healthy and wise.

9. Either this polynomial has degree 2 or it has degree 3 but not both. (n = "This polynomial has degree 2," k = "This polynomial has degree 3")

10. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.
a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.
b. DATAENDFLAG is off but ERROR is not equal to 0.
c. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.
d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

11. In the following sentence, is the word *or* used in its inclusive or exclusive sense? A team wins the playoffs if it wins two games in a row or a total of *three* games.

Write truth tables for the statement forms in 12–15.

12. $\sim p \wedge q$ 13. $\sim(p \wedge q) \vee (p \vee q)$

14. $p \wedge (q \wedge r)$ 15. $p \wedge (\sim q \vee r)$

Determine whether the statement forms in 16–24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

16. $p \vee (p \wedge q)$ and p 17. $\sim(p \wedge q)$ and $\sim p \wedge \sim q$

18. $p \vee \mathbf{t}$ and $\mathbf{t}$ 19. $p \wedge \mathbf{t}$ and p

20. $p \wedge \mathbf{c}$ and $p \vee \mathbf{c}$

21. $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$

*For exercises with blue numbers or letters, solutions are given in Appendix B. The symbol ***H*** indicates that only a hint or a partial solution is given. The symbol ✱ signals that an exercise is more challenging than usual.

22. $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

23. $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$

24. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

Use De Morgan's laws to write negations for the statements in 25–31.

25. Hal is a math major and Hal's sister is a computer science major.

26. Sam is an orange belt and Kate is a red belt.

27. The connector is loose or the machine is unplugged.

28. The units digit of 4^{67} is 4 or it is 6.

29. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

30. The dollar is at an all-time high and the stock market is at a record low.

31. The train is late or my watch is fast.

Assume x is a particular real number and use De Morgan's laws to write negations for the statements in 32–37.

32. $-2 < x < 7$

33. $-10 < x < 2$

34. $x < 2$ or $x > 5$

35. $x \leq -1$ or $x > 1$

36. $1 > x \geq -3$

37. $0 > x \geq -7$

In 38 and 39, imagine that *num_orders* and *num_instock* are particular values, such as might occur during execution of a computer program. Write negations for the following statements.

38. (*num_orders* > 100 and *num_instock* ≤ 500) or *num_instock* < 200

39. (*num_orders* < 50 and *num_instock* > 300) or (50 ≤ *num_orders* < 75 and *num_instock* > 500)

Use truth tables to establish which of the statement forms in 40–43 are tautologies and which are contradictions.

40. $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

41. $(p \wedge \sim q) \wedge (\sim p \vee q)$

42. $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

43. $(\sim p \vee q) \vee (p \wedge \sim q)$

In 44 and 45, determine whether the statements in (a) and (b) are logically equivalent.

44. Assume x is a particular real number.

a. $x < 2$ or it is not the case that $1 < x < 3$.

b. $x \leq 1$ or either $x < 2$ or $x \geq 3$.

45. a. Bob is a double math and computer science major and Ann is a math major, but Ann is not a double math and computer science major.

b. It is not the case that both Bob and Ann are double math and computer science majors, but it is the case that Ann is a math major and Bob is a double math and computer science major.

✶46. In Example 2.1.4, the symbol ⊕ was introduced to denote *exclusive or*, so $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$. Hence the truth table for *exclusive or* is as follows:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

a. Find simpler statement forms that are logically equivalent to $p \oplus p$ and $(p \oplus p) \oplus p$.
b. Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? Justify your answer.
c. Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? Justify your answer.

✶**47.** In logic and in standard English, a double negative is equivalent to a positive. There is one fairly common English usage in which a "double positive" is equivalent to a negative. What is it? Can you think of others?

In 48 and 49 below, a logical equivalence is derived from Theorem 2.1.1. Supply a reason for each step.

48.
$$\begin{aligned}(p \wedge \sim q) \vee (p \wedge q) &\equiv p \wedge (\sim q \vee q) && \text{by } \underline{\text{(a)}}\\ &\equiv p \wedge (q \vee \sim q) && \text{by } \underline{\text{(b)}}\\ &\equiv p \wedge \mathbf{t} && \text{by } \underline{\text{(c)}}\\ &\equiv p && \text{by } \underline{\text{(d)}}\end{aligned}$$

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

49. $(p \vee \sim q) \wedge (\sim p \vee \sim q)$
$$\begin{aligned}&\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p) && \text{by } \underline{\text{(a)}}\\ &\equiv \sim q \vee (p \wedge \sim p) && \text{by } \underline{\text{(b)}}\\ &\equiv \sim q \vee \mathbf{c} && \text{by } \underline{\text{(c)}}\\ &\equiv \sim q && \text{by } \underline{\text{(d)}}\end{aligned}$$

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.

Use Theorem 2.1.1 to verify the logical equivalences in 50–54. Supply a reason for each step.

50. $(p \wedge \sim q) \vee p \equiv p$

51. $p \wedge (\sim q \vee p) \equiv p$

52. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

53. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

54. $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$