Exercise Set 2.2

Rewrite the statements in 1–4 in if-then form.

1. This loop will repeat exactly N times if it does not contain a **stop** or a **go to.**

2. I am on time for work if I catch the 8:05 bus.

3. Freeze or I'll shoot.

4. Fix my ceiling or I won't pay my rent.

Construct truth tables for the statement forms in 5–11.

5. $\sim p \vee q \to \sim q$

6. $(p \vee q) \vee (\sim p \wedge q) \to q$

7. $p \wedge \sim q \to r$

8. $\sim p \vee q \to r$

9. $p \wedge \sim r \leftrightarrow q \vee r$

10. $(p \to r) \leftrightarrow (q \to r)$

11. $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

12. Use the logical equivalence established in Example 2.2.3, $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$, to rewrite the following statement. (Assume that x represents a fixed real number.)

If $x > 2$ or $x < -2$, then $x^2 > 4$.

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.

a. $p \to q \equiv \sim p \vee q$ b. $\sim(p \to q) \equiv p \wedge \sim q$.

H 14. **a.** Show that the following statement forms are all logically equivalent.

$p \to q \vee r$, $p \wedge \sim q \to r$, and $p \wedge \sim r \to q$

b. Use the logical equivalences established in part (a) to rewrite the following sentence in two different ways. (Assume that n represents a fixed integer.)

If n is prime, then n is odd or n is 2.

15. Determine whether the following statement forms are logically equivalent:

$p \to (q \to r)$ and $(p \to q) \to r$

In 16 and 17, write each of the two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words of explanation.

16. If you paid full price, you didn't buy it at Crown Books. You didn't buy it at Crown Books or you paid full price.

17. If 2 is a factor of n and 3 is a factor of n, then 6 is a factor of n. 2 is not a factor of n or 3 is not a factor of n or 6 is a factor of n.

18. Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Include truth tables and a few words of explanation.

If it walks like a duck and it talks like a duck, then it is a duck.

Either it does not walk like a duck or it does not talk like a duck, or it is a duck.

If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

19. True or false? The negation of "If Sue is Luiz's mother, then Ali is his cousin" is "If Sue is Luiz's mother, then Ali is not his cousin."

20. Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

a. If P is a square, then P is a rectangle.
b. If today is New Year's Eve, then tomorrow is January.
c. If the decimal expansion of r is terminating, then r is rational.
d. If n is prime, then n is odd or n is 2.
e. If x is nonnegative, then x is positive or x is 0.
f. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
g. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

21. Suppose that p and q are statements so that $p \to q$ is false. Find the truth values of each of the following:

a. $\sim p \to q$ b. $p \vee q$ c. $q \to p$

H **22.** Write contrapositives for the statements of exercise 20.

H **23.** Write the converse and inverse for each statement of exercise 20.

Use truth tables to establish the truth of each statement in 24–27.

24. A conditional statement is not logically equivalent to its converse.

25. A conditional statement is not logically equivalent to its inverse.

26. A conditional statement and its contrapositive are logically equivalent to each other.

27. The converse and inverse of a conditional statement are logically equivalent to each other.

H **28.** "Do you mean that you think you can find out the answer to it?" said the March Hare.

"Exactly so," said Alice.

"Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."

"Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

—from "A Mad Tea-Party" in *Alice in Wonderland,* by Lewis Carroll

The Hatter is right. "I say what I mean" is not the same thing as "I mean what I say." Rewrite each of these two sentences in if-then form and explain the logical relation between them. (This exercise is referred to in the introduction to Chapter 4.)

If statement forms P and Q are logically equivalent, then $P \leftrightarrow Q$ is a tautology. Conversely, if $P \leftrightarrow Q$ is a tautology, then P and Q are logically equivalent. Use $\leftrightarrow$ to convert each of the logical equivalences in 29–31 to a tautology. Then use a truth table to verify each tautology.

29. $p \to (q \vee r) \equiv (p \wedge \sim q) \to r$

30. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

31. $p \to (q \to r) \equiv (p \wedge q) \to r$

Rewrite each of the statements in 32 and 33 as a conjunction of two if-then statements.

32. This quadratic equation has two distinct real roots if, and only if, its discriminant is greater than zero.

33. This integer is even if, and only if, it equals twice some integer.

Rewrite the statements in 34 and 35 in if-then form in two ways, one of which is the contrapositive of the other.

34. The Cubs will win the pennant only if they win tomorrow's game.

35. Sam will be allowed on Signe's racing boat only if he is an expert sailor.

36. Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired *only if* you major in mathematics or computer science, get a B average or better, and take accounting. You do, in fact, become a math major, get a B^+ average, and take accounting. You return to Prestige Corporation, make a formal application, and are turned down. Did the personnel director lie to you?

Some programming languages use statements of the form "r unless s" to mean that as long as s does not happen, then r will happen. More formally:

> **Definition:** If r and s are statements,
>
> **r unless s** means if $\sim s$ then r.

In 37–39, rewrite the statements in if-then form.

37. Payment will be made on the fifth unless a new hearing is granted.

38. Ann will go unless it rains.

39. This door will not open unless a security code is entered.

Rewrite the statements in 40 and 41 in if-then form.

40. Catching the 8:05 bus is a sufficient condition for my being on time for work.

41. Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

Use the contrapositive to rewrite the statements in 42 and 43 in if-then form in two ways.

42. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

43. Doing homework regularly is a necessary condition for Jim to pass the course.

Note that "a sufficient condition for s is r" means r is a sufficient condition for s and that "a necessary condition for s is r" means r is a necessary condition for s. Rewrite the statements in 44 and 45 in if-then form.

44. A sufficient condition for Jon's team to win the championship is that it win the rest of its games.

45. A necessary condition for this computer program to be correct is that it not produce error messages during translation.

46. "If compound X is boiling, then its temperature must be at least 150°C." Assuming that this statement is true, which of the following must also be true?

a. If the temperature of compound X is at least 150°C, then compound X is boiling.

b. If the temperature of compound X is less than 150°C, then compound X is not boiling.

c. Compound X will boil only if its temperature is at least 150°C.

d. If compound X is not boiling, then its temperature is less than 150°C.

e. A necessary condition for compound X to boil is that its temperature be at least 150°C.

f. A sufficient condition for compound X to boil is that its temperature be at least 150°C.

In 47–50 (a) use the logical equivalences $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\leftrightarrow$, and (b) use the logical equivalence $p \vee q \equiv \sim(\sim p \wedge \sim q)$ to rewrite each statement form using only $\wedge$ and $\sim$.

47. $p \wedge \sim q \to r$

48. $p \vee \sim q \to r \vee q$

49. $(p \to r) \leftrightarrow (q \to r)$

50. $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

51. Given any statement form, is it possible to find a logically equivalent form that uses only $\sim$ and $\wedge$? Justify your answer.