## Exercise Set 2.4

Give the output signals for the circuits in $1-4$ if the input signals are as indicated.
1.

input signals: $P=1 \quad$ and $\quad Q=1$
2.

input signals: $P=1$ and $Q=0$
3.

input signals: $P=1, \quad Q=0, \quad R=0$
4.

input signals: $P=0, \quad Q=0, \quad R=0$
In 5-8, write an input/output table for the circuit in the referenced exercise.
5. Exercise 1
6. Exercise 2
7. Exercise 3
8. Exercise 4

In 9-12, find the Boolean expression that corresponds to the circuit in the referenced exercise.
9. Exercise 1
10. Exercise 2
11. Exercise 3
12. Exercise 4

Construct circuits for the Boolean expressions in 13-17.
13. $\sim P \vee Q$
14. $\sim(P \vee Q)$
15. $P \vee(\sim P \wedge \sim Q)$
16. $(P \wedge Q) \vee \sim R$
17. $(P \wedge \sim Q) \vee(\sim P \wedge R)$

For each of the tables in 18-21, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table.
18.

| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{R}$ | $\boldsymbol{S}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

19. 

| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{R}$ | $\boldsymbol{S}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

20. 

| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{R}$ | $\boldsymbol{S}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

21. 

| $\boldsymbol{P}$ | $\boldsymbol{Q}$ | $\boldsymbol{R}$ | $\boldsymbol{S}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

22. Design a circuit to take input signals $P, Q$, and $R$ and output a 1 if, and only if, $P$ and $Q$ have the same value and $Q$ and $R$ have opposite values.
23. Design a circuit to take input signals $P, Q$, and $R$ and output a 1 if, and only if, all three of $P, Q$, and $R$ have the same value.
24. The lights in a classroom are controlled by two switches: one at the back and one at the front of the room. Moving either switch to the opposite position turns the lights off if they are on and on if they are off. Assume the lights have been installed so that when both switches are in the down position, the lights are off. Design a circuit to control the switches.
25. An alarm system has three different control panels in three different locations. To enable the system, switches in at least two of the panels must be in the on position. If fewer than two are in the on position, the system is disabled. Design a circuit to control the switches.
Use the properties listed in Theorem 2.1.1 to show that each pair of circuits in 26-29 have the same input/output table. (Find the Boolean expressions for the circuits and show that they are ${ }^{-}$ logically equivalent when regarded as statement forms.)
26. a.

b.

27. a.

b.

28. a.

b.

29. a.

b.


For the circuits corresponding to the Boolean expressions in each of 30 and 31 there is an equivalent circuit with at most two logic gates. Find such a circuit.
30. $(P \wedge Q) \vee(\sim P \wedge Q) \vee(\sim P \wedge \sim Q)$
31. $(\sim P \wedge \sim Q) \vee(\sim P \wedge Q) \vee(P \wedge \sim Q)$
32. The Boolean expression for the circuit in Example 2.4.5 is

$$
(P \wedge Q \wedge R) \vee(P \wedge \sim Q \wedge R) \vee(P \wedge \sim Q \wedge \sim R)
$$

(a disjunctive normal form). Find a circuit with at most three logic gates that is equivalent to this circuit.
33. a. Show that for the Sheffer stroke |,

$$
P \wedge Q \equiv(P \mid Q) \mid(P \mid Q)
$$

b. Use the results of Example 2.4.7 and part (a) above to write $P \wedge(\sim Q \vee R)$ using only Sheffer strokes.
34. Show that the following logical equivalences hold for the Peirce arrow $\downarrow$, where $P \downarrow Q \equiv \sim(P \vee Q)$.
a. $\sim P \equiv P \downarrow P$
b. $P \vee Q \equiv(P \downarrow Q) \downarrow(P \downarrow Q)$
c. $P \wedge Q \equiv(P \downarrow P) \downarrow(Q \downarrow Q)$

H d. Write $P \rightarrow Q$ using Peirce arrows only.
e. Write $P \leftrightarrow Q$ using Peirce arrows only.

