Exercise Set 3.2

1. Which of the following is a negation for "All discrete mathematics students are athletic"? More than one answer may be correct.
   a. There is a discrete mathematics student who is nonathletic.
   b. All discrete mathematics students are nonathletic.
   c. There is an athletic person who is a discrete mathematics student.
   d. No discrete mathematics students are athletic.
   e. Some discrete mathematics students are nonathletic.
   f. No athletic people are discrete mathematics students.

2. Which of the following is a negation for "All dogs are loyal"? More than one answer may be correct.
   a. All dogs are disloyal. b. No dogs are loyal.
   c. Some dogs are disloyal. d. Some dogs are loyal.
   e. There is a disloyal animal that is not a dog.
   f. There is a dog that is disloyal.
   g. No animals that are not dogs are loyal.
   h. Some animals that are not dogs are loyal.

3. Write a formal negation for each of the following statements:
   a. $\forall$ fish x, x has gills.
   b. $\forall$ computers c, c has a CPU.
   c. $\exists$ a movie m such that m is over 6 hours long.
   d. $\exists$ a band b such that b has won at least 10 Grammy awards.

4. Write an informal negation for each of the following statements. Be careful to avoid negations that are ambiguous.
   a. All dogs are friendly.
   b. All people are happy.
   c. Some suspicions were substantiated.
   d. Some estimates are accurate.

5. Write a negation for each of the following statements.
   a. Any valid argument has a true conclusion.
   b. Every real number is positive, negative, or zero.

6. Write a negation for each of the following statements.
   a. Sets A and B do not have any points in common.
   b. Towns P and Q are not connected by any road on the map.

7. Informal language is actually more complex than formal language. For instance, the sentence "There are no orders from store A for item B" contains the words *there are*. Is the statement existential? Write an informal negation for the statement, and then write the statement formally using quantifiers and variables.

8. Consider the statement "There are no simple solutions to life's problems." Write an informal negation for the statement, and then write the statement formally using quantifiers and variables.

Write a negation for each statement in 9 and 10.

9. $\forall$ real numbers x, if $x > 3$ then $x^2 > 9$.

10. $\forall$ computer programs P, if P compiles without error messages, then P is correct.

In each of 11–14 determine whether the proposed negation is correct. If it is not, write a correct negation.

11. *Statement:* The sum of any two irrational numbers is irrational.
Proposed negation: The sum of any two irrational numbers is rational.

12. *Statement:* The product of any irrational number and any rational number is irrational.
Proposed negation: The product of any irrational number and any rational number is rational.

13. *Statement:* For all integers n, if n^2 is even then n is even.
Proposed negation: For all integers n, if n^2 is even then n is not even.

14. *Statement:* For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 = x_2$.
Proposed negation: For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 \neq x_2$.

15. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
   a. $\forall x \in D$, if x is odd then $x > 0$.
   b. $\forall x \in D$, if x is less than 0 then x is even.
   c. $\forall x \in D$, if x is even then $x \leq 0$.
   d. $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
   e. $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

In 16–23, write a negation for each statement.

16. $\forall$ real numbers x, if $x^2 \geq 1$ then $x > 0$.

17. $\forall$ integers d, if $6/d$ is an integer then $d = 3$.

18. $\forall x \in \mathbf{R}$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.

19. $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

20. $\forall$ integers a, b and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even.

21. $\forall$ integers n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

22. If the square of an integer is odd, then the integer is odd.

23. If a function is differentiable then it is continuous.

24. Rewrite the statements in each pair in if-then form and indicate the logical relationship between them.
   a. All the children in Tom's family are female.
   All the females in Tom's family are children.
   b. All the integers that are greater than 5 and end in 1, 3, 7, or 9 are prime.
   All the integers that are greater than 5 and are prime end in 1, 3, 7, or 9.

25. Each of the following statements is true. In each case write the converse of the statement, and give a counterexample showing that the converse is false.
   a. If n is any prime number that is greater than 2, then $n + 1$ is even.
   b. If m is any odd integer, then $2m$ is even.
   c. If two circles intersect in exactly two points, then they do not have a common center.

In 26–33, for each statement in the referenced exercise write the converse, inverse, and contrapositive. Indicate as best as you can which among the statement, its converse, its inverse, and its contrapositive are true and which are false. Give a counterexample for each that is false.

26. Exercise 16 27. Exercise 17

28. Exercise 18 29. Exercise 19

30. Exercise 20 31. Exercise 21

32. Exercise 22 33. Exercise 23

34. Write the contrapositive for each of the following statements.
   a. If n is prime, then n is not divisible by any prime number between 1 and $\sqrt{n}$ strictly. (Assume that n is a fixed integer that is greater than 1.)
   b. If A and B do not have any elements in common, then they are disjoint. (Assume that A and B are fixed sets.)

35. Give an example to show that a universal conditional statement is not logically equivalent to its inverse.

★36. If $P(x)$ is a predicate and the domain of x is the set of all real numbers, let R be "$\forall x \in \mathbf{Z}, P(x)$," let S be "$\forall x \in \mathbf{Q}, P(x)$," and let T be "$\forall x \in \mathbf{R}, P(x)$."
   a. Find a definition for $P(x)$ (but do not use "$x \in \mathbf{Z}$") so that R is true and both S and T are false.
   b. Find a definition for $P(x)$ (but do not use "$x \in \mathbf{Q}$") so that both R and S are true and T is false.

37. Consider the following sequence of digits: 0204. A person claims that all the 1's in the sequence are to the left of all the 0's in the sequence. Is this true? Justify your answer. (*Hint:* Write the claim formally and write a formal negation for it. Is the negation true or false?)

38. True or false? All occurrences of the letter u in *Discrete Mathematics* are lowercase. Justify your answer.

Rewrite each statement of 39–42 in if-then form.

39. Earning a grade of C− in this course is a sufficient condition for it to count toward graduation.

40. Being divisible by 8 is a sufficient condition for being divisible by 4.

41. Being on time each day is a necessary condition for keeping this job.

42. Passing a comprehensive exam is a necessary condition for obtaining a master's degree.

Use the facts that the negation of a $\forall$ statement is a $\exists$ statement and that the negation of an if-then statement is an *and* statement to rewrite each of the statements 43–46 without using the word *necessary* or *sufficient*.

43. Being divisible by 8 is not a necessary condition for being divisible by 4.

44. Having a large income is not a necessary condition for a person to be happy.

45. Having a large income is not a sufficient condition for a person to be happy.

46. Being a polynomial is not a sufficient condition for a function to have a real root.

47. The computer scientists Richard Conway and David Gries once wrote:

> The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.

Rewrite this statement without using the words *necessary* or *sufficient*.

48. A frequent-flyer club brochure states, "You may select among carriers only if they offer the same lowest fare." Assuming that "only if" has its formal, logical meaning, does this statement guarantee that if two carriers offer the same lowest fare, the customer will be free to choose between them? Explain.