Exercise Set 3.4

1. Let the following law of algebra be the first statement of an argument: For all real numbers a and b,

$$(a+b)^2 = a^2 + 2ab + b^2.$$

Suppose each of the following statements is, in turn, the second statement of the argument. Use universal instantiation or universal modus ponens to write the conclusion that follows in each case.

a. $a = x$ and $b = y$ are particular real numbers.
b. $a = f_i$ and $b = f_j$ are particular real numbers.
c. $a = 3u$ and $b = 5v$ are particular real numbers.
d. $a = g(r)$ and $b = g(s)$ are particular real numbers.
e. $a = \log(t_1)$ and $b = \log(t_2)$ are particular real numbers.

Use universal instantiation or universal modus ponens to fill in valid conclusions for the arguments in 2–4.

2. If an integer n equals $2 \cdot k$ and k is an integer, then n is even.
0 equals $2 \cdot 0$ and 0 is an integer.
$\therefore$ ________________.

3. For all real numbers a, b, c, and d, if $b \neq 0$ and $d \neq 0$, then $a/b + c/d = (ad + bc)/bd$.
$a = 2$, $b = 3$, $c = 4$, and $d = 5$ are particular real numbers such that $b \neq 0$ and $d \neq 0$.
$\therefore$ ________________.

4. $\forall$ real numbers r, a, and b, if r is positive, then $(r^a)^b = r^{ab}$.
$r = 3$, $a = 1/2$, and $b = 6$ are particular real numbers such that r is positive.
$\therefore$ ________________.

Use universal modus tollens to fill in valid conclusions for the arguments in 5 and 6.

5. All irrational numbers are real numbers
$\frac{1}{0}$ is not a real number.
$\therefore$ ________________.

6. If a computer program is correct, then compilation of the program does not produce error messages.
Compilation of this program produces error messages.
$\therefore$ ________________.

Some of the arguments in 7–18 are valid by universal modus ponens or universal modus tollens; others are invalid and exhibit the converse or the inverse error. State which are valid and which are invalid. Justify your answers.

7. All healthy people eat an apple a day.
Keisha eats an apple a day.
$\therefore$ Keisha is a healthy person.

8. All freshmen must take writing.
Caroline is a freshman.
∴ Caroline must take writing.

9. All healthy people eat an apple a day.
Herbert is not a healthy person.
∴ Herbert does not eat an apple a day.

10. If a product of two numbers is 0, then at least one of the numbers is 0.
For a particular number x, neither $(2x + 1)$ nor $(x - 7)$ equals 0.
∴ The product $(2x + 1)(x - 7)$ is not 0.

11. All cheaters sit in the back row.
Monty sits in the back row.
∴ Monty is a cheater.

12. All honest people pay their taxes.
Darth is not honest.
∴ Darth does not pay his taxes.

13. For all students x, if x studies discrete mathematics, then x is good at logic.
Tarik studies discrete mathematics.
∴ Tarik is good at logic.

14. If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
∴ This program is correct.

15. Any sum of two rational numbers is rational.
The sum $r + s$ is rational.
∴ The numbers r and s are both rational.

16. If a number is even, then twice that number is even.
The number $2n$ is even, for a particular number n.
∴ The particular number n is even.

17. If an infinite series converges, then the terms go to 0.
The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.
∴ The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

18. If an infinite series converges, then its terms go to 0.
The terms of the infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ do not go to 0.
∴ The infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ does not converge.

19. Rewrite the statement "No good cars are cheap" in the form "$\forall x$, if $P(x)$ then $\sim Q(x)$." Indicate whether each of the following arguments is valid or invalid, and justify your answers.

a. No good car is cheap.
A Rimbaud is a good car.
∴ A Rimbaud is not cheap.

b. No good car is cheap.
A Simbaru is not cheap.
∴ A Simbaru is a good car.

c. No good car is cheap.
A VX Roadster is cheap.
∴ A VX Roadster is not good.

d. No good car is cheap.
An Omnex is not a good car.
∴ An Omnex is cheap.

20. a. Use a diagram to show that the following argument can have true premises and a false conclusion.

All dogs are carnivorous.
Aaron is not a dog.
∴ Aaron is not carnivorous.

b. What can you conclude about the validity or invalidity of the following argument form? Explain how the result from part (a) leads to this conclusion.

$\forall x$, if $P(x)$ then $Q(x)$.
$\sim P(a)$ for a particular a.
$\therefore \sim Q(a)$.

Indicate whether the arguments in 21–27 are valid or invalid. Support your answers by drawing diagrams.

21. All people are mice.
All mice are mortal.
∴ All people are mortal.

22. All discrete mathematics students can tell a valid argument from an invalid one.
All thoughtful people can tell a valid argument from an invalid one.
∴ All discrete mathematics students are thoughtful.

23. All teachers occasionally make mistakes.
No gods ever make mistakes.
∴ No teachers are gods.

24. No vegetarians eat meat.
All vegans are vegetarian.
∴ No vegans eat meat.

25. No college cafeteria food is good.
No good food is wasted.
∴ No college cafeteria food is wasted.

26. All polynomial functions are differentiable.
All differentiable functions are continuous.
∴ All polynomial functions are continuous.

27. [Adapted from Lewis Carroll.]
Nothing intelligible ever puzzles *me*.
Logic puzzles me.
∴ Logic is unintelligible.

In exercises 28–32, reorder the premises in each of the arguments to show that the conclusion follows as a valid consequence from the premises. It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives. Exercises 28–30 refer to the kinds of Tarski worlds discussed in Example 3.1.13 and 3.3.1. Exercises 31 and 32 are adapted from *Symbolic Logic* by Lewis Carroll.*

28. 1. Every object that is to the right of all the blue objects is above all the triangles.
2. If an object is a circle, then it is to the right of all the blue objects.
3. If an object is not a circle, then it is not gray.
∴ All the gray objects are above all the triangles.

29. 1. All the objects that are to the right of all the triangles are above all the circles.
2. If an object is not above all the black objects, then it is not a square.
3. All the objects that are above all the black objects are to the right of all the triangles.
∴ All the squares are above all the circles.

30. 1. If an object is above all the triangles, then it is above all the blue objects.
2. If an object is not above all the gray objects, then it is not a square.
3. Every black object is a square.
4. Every object that is above all the gray objects is above all the triangles.
∴ If an object is black, then it is above all the blue objects.

31. 1. I trust every animal that belongs to me.
2. Dogs gnaw bones.
3. I admit no animals into my study unless they will beg when told to do so.
4. All the animals in the yard are mine.
5. I admit every animal that I trust into my study.
6. The only animals that are really willing to beg when told to do so are dogs.
∴ All the animals in the yard gnaw bones.

32. 1. When I work a logic example without grumbling, you may be sure it is one I understand.
2. The arguments in these examples are not arranged in regular order like the ones I am used to.
3. No easy examples make my head ache.
4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.
5. I never grumble at an example unless it gives me a headache.
∴ These examples are not easy.

*Lewis Carroll, *Symbolic Logic* (New York: Dover, 1958), pp. 118, 120, 123.

In 33 and 34 a single conclusion follows when all the given premises are taken into consideration, but it is difficult to see because the premises are jumbled up. Reorder the premises to make it clear that a conclusion follows logically, and state the valid conclusion that can be drawn. (It may be helpful to rewrite some of the statements in if-then form and to replace some statements by their contrapositives.)

33. 1. No birds except ostriches are at least 9 feet tall.
2. There are no birds in this aviary that belong to anyone but me.
3. No ostrich lives on mince pies.
4. I have no birds less than 9 feet high.

34. 1. All writers who understand human nature are clever.
2. No one is a true poet unless he can stir the human heart.
3. Shakespeare wrote *Hamlet*.
4. No writer who does not understand human nature can stir the human heart.
5. None but a true poet could have written *Hamlet*.

✶ 35. Derive the validity of universal modus tollens from the validity of universal instantiation and modus tollens.

✶ 36. Derive the validity of universal form of part(a) of the elimination rule from the validity of universal instantiation and the valid argument called elimination in Section 2.3.