Exercise Set 9.6

1. a. According to Theorem 9.6.1, how many 5-combinations with repetition allowed can be chosen from a set of three elements?
   b. List all of the 5-combinations that can be chosen with repetition allowed from $\{1, 2, 3\}$.

2. a. According to Theorem 9.6.1, how many multisets of size four can be chosen from a set of three elements?
   b. List all of the multisets of size four that can be chosen from the set $\{x, y, z\}$.

3. A bakery produces six different kinds of pastry, one of which is eclairs. Assume there are at least 20 pastries of each kind.
   a. How many different selections of twenty pastries are there?
   b. How many different selections of twenty pastries are there if at least three must be eclairs?
   c. How many different selections of twenty pastries contain at most two eclairs?

4. A camera shop stocks eight different types of batteries, one of which is type A7b. Assume there are at least 30 batteries of each type.
   a. How many ways can a total inventory of 30 batteries be distributed among the eight different types?
   b. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory must include at least four A76 batteries?
   c. How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory includes at most three A7b batteries?

5. If n is a positive integer, how many 4-tuples of integers from 1 through n can be formed in which the elements of the 4-tuple are written in increasing order but are not necessarily distinct? In other words, how many 4-tuples of integers (i, j, k, m) are there with $1 \le i \le j \le k \le m \le n$?

6. If n is a positive integer, how many 5-tuples of integers from 1 through n can be formed in which the elements of the 5-tuple are written in decreasing order but are not necessarily distinct? In other words, how many 5-tuples of integers (h, i, j, k, m) are there with $n \ge h \ge i \ge j \ge k \ge m \ge 1$?

7. Another way to count the number of nonnegative integral solutions to an equation of the form $x_1 + x_2 + \cdots + x_n = m$ is to reduce the problem to one of finding the number of n-tuples $(y_1, y_2, \ldots, y_n)$ with $0 \le y_1 \le y_2 \le \cdots \le y_n \le m$. The reduction results from letting $y_i = x_1 + x_2 + \cdots + x_i$ for each $i = 1, 2, \ldots, n$. Use this approach to derive a general formula for the number of nonnegative integral solutions to $x_1 + x_2 + \cdots + x_n = m$.

8 and 9, how many times will the innermost loop be iterated when the algorithm segment is implemented and run? Assume n, m, k, and j are positive integers.

```
for m := 1 to n
    for k := 1 to m
        for j := 1 to k
            for i := 1 to j
                [Statements in the body of the inner loop,
                none containing branching statements that
                lead outside the loop]
            next i
        next j
    next k
next m
```

```
for k := 1 to n
    for j := k to n
        for i := j to n
                [Statements in the body of the inner loop,
                none containing branching statements that
                lead outside the loop]
            next i
        next j
    next k
```

10–14, find how many solutions there are to the given equation that satisfy the given condition.

10. $x_1 + x_2 + x_3 = 20$, each x_i is a nonnegative integer.

11. $x_1 + x_2 + x_3 = 20$, each x_i is a positive integer.

12. $y_1 + y_2 + y_3 + y_4 = 30$, each y_i is a nonnegative integer.

13. $y_1 + y_2 + y_3 + y_4 = 30$, each y_i is an integer that is at least 2.

14. $a + b + c + d + e = 500$, each of a, b, c, d, and e is an integer that is at least 10.

15. For how many integers from 1 through 99,999 is the sum of their digits equal to 10?

16. Consider the situation in Example 9.6.2.
    a. Suppose the store has only six cans of lemonade but at least 15 cans of each of the other four types, of soft drink. In how many different ways can five cans of soft drink be selected?
    b. Suppose that the store has only five cans of root beer and only six cans of lemonade but at least 15 cans of each of the other three types of soft drink. In how many different ways can five cans of soft drink be selected?

***H*17.** a. A store sells 8 kinds of balloons with at least 30 of each kind. How many different combinations of 30 balloons can be chosen?
    b. If the store has only 12 red balloons but at least 30 of each other kind of balloon, how many combinations of balloons can be chosen?
    c. If the store has only 8 blue balloons but at least 30 of each other kind of balloon, how many combinations of balloons can be chosen?
    d. If the store has only 12 red balloons and only 8 blue balloons but at least 30 of each other kind of balloon, how many combinations of balloons can be chosen?

18. A large pile of coins consists of pennies, nickels, dimes, and quarters.
    a. How many different collections of 30 coins can be chosen if there are at least 30 of each kind of coin?
    b. If the pile contains only 15 quarters but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?
    c. If the pile contains only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?
    d. If the pile contains only 15 quarters and only 20 dimes but at least 30 of each other kind of coin, how many collections of 30 coins can be chosen?

***H*19.** Suppose the bakery in exercise 3 has only ten eclairs but has at least twenty of each of the other kinds of pastry.
    a. How many different selections of twenty pastries are there?
    b. Suppose in addition to having only ten eclairs, the bakery has only eight napoleon slices. How many different selections of twenty pastries are there?

20. Suppose the camera shop in exercise 4 can obtain at most ten A76 batteries but can get at least 30 of each of the other types.
    a. How many ways can a total inventory of 30 batteries be distributed among the eight different types?
    b. Suppose that in addition to being able to obtain only ten A76 batteries, the store can get only six of type D303. How many ways can a total inventory of 30 batteries be distributed among the eight different types?

21. Observe that the number of columns in the trace table for Example 9.6.4 can be expressed as the sum

$$1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + \cdots + n).$$

Explain why this is so, and show how this sum simplifies to the same expression given in the solution of Example 9.6.4. *Hint*: Use a formula from the exercise set for Section 5.2.