

Exercise Set 9.8

- In any sample space S , what is $P(\emptyset)$?
- Suppose A , B , and C are mutually exclusive events in a sample space S , $A \cup B \cup C = S$, and A and B have probabilities 0.3 and 0.5, respectively.
 - What is $P(A \cup B)$?
 - What is $P(C)$?
- Suppose A and B are mutually exclusive events in a sample space S , C is another event in S , $A \cup B \cup C = S$, and A and B have probabilities 0.4 and 0.2, respectively.
 - What is $P(A \cup B)$?
 - Is it possible that $P(C) = 0.2$? Explain.
- Suppose A and B are events in a sample space S with probabilities 0.8 and 0.7, respectively. Suppose also that $P(A \cap B) = 0.6$. What is $P(A \cup B)$?
- Suppose A and B are events in a sample space S and suppose that $P(A) = 0.6$, $P(B^c) = 0.4$, and $P(A \cap B) = 0.2$. What is $P(A \cup B)$?
- Suppose U and V are events in a sample space S and suppose that $P(U^c) = 0.3$, $P(V) = 0.6$, and $P(U^c \cup V^c) = 0.4$. What is $P(U \cup V)$?
- Suppose a sample space S consists of three outcomes: 0, 1, and 2. Let $A = \{0\}$, $B = \{1\}$, and $C = \{2\}$, and suppose $P(A) = 0.4$, and $P(B) = 0.3$. Find each of the following:
 - $P(A \cup B)$
 - $P(C)$
 - $P(A \cup C)$
 - $P(A^c)$
 - $P(A^c \cap B^c)$
 - $P(A^c \cup B^c)$
- Redo exercise 7 assuming that $P(A) = 0.5$ and $P(B) = 0.4$.
- Let A and B be events in a sample space S , and let $C = S - (A \cup B)$. Suppose $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$. Find each of the following:

- $P(A \cup B)$
- $P(C)$
- $P(A^c)$
- $P(A^c \cap B^c)$
- $P(A^c \cup B^c)$
- $P(B^c \cap C)$

- Redo exercise 9 assuming that $P(A) = 0.7$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$.
- H 11.** Prove that if S is any sample space and U and V are events in S with $U \subseteq V$, then $P(U) \leq P(V)$.
- H 12.** Prove that if S is any sample space and U and V are any events in S , then $P(V - U) = P(V) - P(U \cap V)$.
- H 13.** Use the axioms for probability and mathematical induction to prove that for all integers $n \geq 2$, if $A_1, A_2, A_3, \dots, A_n$ are any mutually disjoint events in a sample space S , then

$$P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n) = \sum_{k=1}^n P(A_k).$$

- A lottery game offers \$2 million to the grand prize winner, \$20 to each of 10,000 second prize winners, and \$4 to each of 50,000 third prize winners. The cost of the lottery is \$2 per ticket. Suppose that 1.5 million tickets are sold. What is the expected gain or loss of a ticket?
- A company sends millions of people an entry form for a sweepstakes accompanied by an order form for magazine subscriptions. The first, second, and third prizes are \$10,000,000, \$1,000,000, and \$50,000, respectively. In order to qualify for a prize, a person is not required to order any magazines but has to spend 60 cents to mail back the entry form. If 30 million people qualify by sending back their entry forms, what is a person's expected gain or loss?
- An urn contains four balls numbered 2, 2, 5, and 6. If a person selects a set of two balls at random, what is the expected value of the sum of the numbers on the balls?

17. An urn contains five balls numbered 1, 2, 2, 8, and 8. If a person selects a set of two balls at random, what is the expected value of the sum of the numbers on the balls?
18. An urn contains five balls numbered 1, 2, 2, 8, and 8. If a person selects a set of three balls at random, what is the expected value of the sum of the numbers on the balls?
19. When a pair of balanced dice are rolled and the sum of the numbers showing face up is computed, the result can be any number from 2 to 12, inclusive. What is the expected value of the sum?
- H 20.** Suppose a person offers to play a game with you. In this game, when you draw a card from a standard 52-card deck, if the card is a face card you win \$3, and if the card is anything else you lose \$1. If you agree to play the game, what is your expected gain or loss?
21. A person pays \$1 to play the following game: The person tosses a fair coin four times. If no heads occur, the person pays an additional \$2, if one head occurs, the person pays an additional \$1, if two heads occur, the person just loses the initial dollar, if three heads occur, the person wins \$3, and if four heads occur, the person wins \$4. What is the person's expected gain or loss?
- H 22.** A fair coin is tossed until either a head comes up or four tails are obtained. What is the expected number of tosses?
- H 23.** A gambler repeatedly bets that a die will come up 6 when rolled. Each time the die comes up 6, the gambler wins \$1; each time it does not, the gambler loses \$1. He will quit playing either when he is ruined or when he wins \$300. If P_n is the probability that the gambler is ruined when he begins play with \$ n , then $P_{k-1} = \frac{1}{6}P_k + \frac{5}{6}P_{k-2}$ for all integers k with $2 \leq k \leq 300$. Also $P_0 = 1$ and $P_{300} = 0$. Find an explicit formula for P_n and use it to calculate P_{20} . (Exercise 33 in Section 9.9 asks you to derive the recurrence relation.)