

3. (5 pts) Carefully prove: the difference between any two odd integers is even.

let  $n$  and  $m$  be odd integers

$$n = 2k_n + 1 \quad \text{and} \quad m = 2k_m + 1$$

$$n - m = 2k$$

$$= (2k_n + 1) - (2k_m + 1) = 2k$$

$$= 2(k_n - k_m) = 2k \quad \text{let } k_n - k_m = k$$

$$= 2k = 2k$$

$$\therefore n - m = 2k$$

Q.E.D.

definition of odd

definition of even

closure of constant / B.A.

definition of even

*K and S confusion*

4. (5 pts) Carefully prove: the quotient of any two nonzero rational numbers is rational

6. (2 pts) Show that  $7.21212121\dots$  is a rational number.

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Whenever there is a repeating pattern, as in  $.21212121\dots$ , the number is rational.

10. (5 pts) Prove that if  $n$  is any integer that is not divisible by 2 or 3, then  $n^2 \pmod{12} = 1$ .

$$n \pmod{d} = r$$

def of mod

$$n = qd + r$$

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Quotient remainder theorem

$$5 \quad 7 \quad 11 \quad 13 \quad 17 \quad 19 \quad 23 \quad 29$$

$$n^2 = 12q + 1$$

substitution

*Know and show confusion*

$$\text{let } n = 5$$

substitution of  $a$  number not divisible by 2 or 3

*Arguing from Example*

$$5^2 = 12q + 1$$

Basic Algebra

$$25 = 12q + 1$$

Basic Algebra

$$24 = 12q$$

Basic Algebra

$$q = 2$$

Basic Algebra

using the Quotient remainder theorem and plugging in a number not divisible by 2 or 3 we obtained  $q$  that divided evenly into  $n$ .

9. (5 pts) Prove or disprove:

For all real numbers  $x$  and  $y$ ,  $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

Let  $x$  and  $y$  be any two real numbers. Also, let  $\lceil x \rceil = m$  and  $\lceil y \rceil = n$  where  $m$  and  $n$  are integers

$\lceil xy \rceil = mn$  by substitution

$mn - 1 < \lceil xy \rceil \leq mn$  by def of ceiling

$mn < xy + 1 \leq mn + 1$  by basic algebra.

therefore  $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

*Known and Show  
Confusion*

Q.E.D

10. (5 pts) Prove that if  $n$  is any integer that is not divisible by 2 or 3, then  $n^2 \bmod 12 = 1$ .

Let  $n$  be any integer that is not divisible by 2 or 3

Case 1:  $n$  is not divisible by 2

def. of  
divisibility

$$\rightarrow n = 2q + r \text{ for some integers } q \text{ and } r \text{ and } 0 \leq r < 2 \text{ of}$$

$$\rightarrow n^2 = 12q + 1 = n^2 \bmod 12 = 1 \quad \text{J2C}$$

$$2q + r = 12q + 1$$

$$\text{then } r = 10q + 1$$

since  $r > 1$  then  $n^2 \bmod 12 \neq 1$

Case 2:  $n$  is not divisible by 3

$$n = 3q + r \text{ for some integers } q \text{ and } r \text{ and } 0 \leq r < 3$$

$$n^2 = 12q + 1$$

J2C

$$3q + r = 12q + 1$$

$$r = 9q + 1 \text{ then } n^2 \bmod 12 \neq 1$$

therefore  $r > 1$  then  $n^2 \bmod 12 = 1$

Case 3:  $n$  is divisible by 2

$$n = 2q + 0$$

$$n^2 = 12q + 0$$

$$2q = 12q + 1$$

9. (5 pts) Prove or disprove:

For all real numbers  $x$  and  $y$ ,  $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

Counter Example: let  $x = 0$   
let  $y = 1$

$$\lceil 0 \times 1 \rceil = \lceil 0 \rceil \lceil 1 \rceil$$

$$\lceil 0 \rceil = 1 \times 2$$

$$1 \neq 2$$