#25 Theorem:

The difference of any even integer minus any odd integer is odd.

Proof:

Suppose m is any even integer and n is any odd integer. By definition of even, $m = 2r$ and by definition of odd, $n = 2s + 1$ for some integers r and s.

Then $m - n = 2r - (2s + 1)$ — by substitution

$m - n = 2r - 2s - 1$ — Basic Algebra

$m - n = 2(r - s - 1) + 1$ — Basic Algebra

Let $t = r - s - 1 \in \mathbb{Z}$ — Closure of integers in $\mathbb{Z}$

$m - n = 2t + 1$ — definition of odd.

Therefore $m - n$ is odd.