

#27 The sum of any two integers is even

Proof:

Let m and n be any two odd integers

$$m = 2r + 1$$

Def. odd integers

for some $r \in \mathbb{Z}$

$$n = 2s + 1$$

Def. odd integers

for some $s \in \mathbb{Z}$

$$m+n = (2r+1) + (2s+1)$$

substitution

$$= 2r + 2s + 2$$

Basic Algebra

$$= 2(r+s+1)$$

Basic Algebra

$$\text{Let } k = (r+s+1) \in \mathbb{Z}$$

closure of integers over addition

$$m+n = 2k$$

def. of even integers

$\therefore m+n = \text{an even integer}$

Q.E.D