

28) For all integers  $n$ , if  $n$  is odd, then  $n^2$  is odd

$$n = \text{odd} \quad n = 2k+1$$

$$\begin{aligned}n^2 &= (2k+1)(2k+1) \\&= 4k^2 + 4k + 1 \\&= 4k(k+1) + 1 \\&= (2k) \cdot 2(k+1) + 1 \\&= 2k \cdot 2k + 2 + 1\end{aligned}$$

$$\text{even} = 2k$$

$$\begin{aligned}(2k)(2k) &= 4k^2 \\&= 2(2k^2) \quad r = 2k^2 \\ \text{even} &= 2r\end{aligned}$$

$$n^2 = \text{even} + 3$$

$$2k = \text{even} \quad 2k+1 = \text{odd}$$

$$\begin{aligned}(2k+1) - (2k) &= 1 \\ \text{odd} - \text{even} &= \text{odd} \\ \text{odd} &= \text{odd} + \text{even}\end{aligned}$$

$$n^2 = (2k)(2k) + 3 = \text{odd}$$