

29. For all integers n , if n is odd then $3n + 5$ is even.

Proof:

Let n be an arbitrary odd integer

$$n = 2r + 1 \text{ for some } r \in \mathbb{Z}$$

← Definition of odd

$$3n + 5 = 3(2r + 1) + 5$$

← Substitution

$$= 6r + 3 + 5$$

← Distributive law (basic algebra)

$$= 6r + 8$$

← Addition (basic algebra)

$$= 2(3r + 4)$$

← Distributive law (basic algebra)

$$\text{Let } s = 3r + 4$$

then s is an integer

← Sums, differences, and products of integers are integers

$$\text{therefore } 3n + 5 = 2s$$

← Definition of even

Q. E. D.