

3) If k is any odd integer and m is any even integer then $k^2 + m^2$ is odd

proof: let k be any odd integer and m be any even integer such that

$$k = 2n + 1$$

by defn odd

$$m = 2n$$

by defn even

then

$$k^2 + m^2 = (2n+1)^2 + (2n)^2$$

BA

$$= 4n^2 + 4n + 1 + 4n^2$$

$$= 8n^2 + 4n + 1$$

BA

$$= 4n(2n+1) + 1$$

BA

$$= 2(2n)(2n+1) + 1$$

$$= 2mk + 1$$

$$\text{let } t = mk$$

where $t = 2n+1$ by defn odd \square

and $t \in \mathbb{Z}$

therefore $k^2 + m^2$ is odd

$$k^2 + m^2 = 2t + 1 \quad \text{by defn odd}$$

QED