

#37

- There exist an integer k such that $k \geq 4$ and $2k^2 - 5k + 2$ is prime.
- \exists an integer $k \mid k \geq 4$ and $2k^2 - 5k + 2$ is prime.
- ↳ Showing that this statement is false is equivalent to showing that its negation is true.

negation $\rightarrow \forall$ integers k such that $k \geq 4$ and $2k^2 - 5k + 2$ is not prime.

$$\begin{aligned} 2k^2 - 5k + 2 &= 2k^2 - 4k - k + 2 \\ &= 2k(k-2) - 1(k-2) \\ &= (2k-1)(k-2) \end{aligned}$$

Basic Algebra.

$$\begin{aligned} 1 &< 2k-1 < 2k^2 - 5k + 2 \\ 1 &< k-2 < 2k^2 - 5k + 2 \end{aligned}$$

$(2k-1)$ and $(k-2)$ are positive integers greater than 1 and smaller than $2k^2 - 5k + 2$ since $k \geq 4$.

n is prime $\iff \forall$ positive integers r and s , if $n = r \cdot s$ then $r = 1$ or $s = 1$.

Definition of prime numbers.

$$\begin{aligned} &2k^2 - 5k + 2 \\ &(2k-1)(k-2) \end{aligned}$$

Product of two positive integers greater than 1.

$2k^2 - 5k + 2$ is not a prime.

$\therefore \exists$ an integer k such that $k \geq 4$ and $2k^2 - 5k + 2$ is not prime. \square E.D.