

Pg 102 #43

Theorem: The product of any two odd integers is odd

proof: let  $m$  and  $n$  be any two odd integers

$$m = 2r + 1 \quad \text{for some } r \in \mathbb{Z} \quad \text{Def. of odd}$$

$$n = 2s + 1 \quad \text{for some } s \in \mathbb{Z} \quad \text{Def. of odd}$$

$$mn = (2r + 1)(2s + 1) \quad \text{Basic Algebra}$$

$$mn = 4rs + 2r + 2s + 1 \quad \text{Basic Algebra}$$

$$mn = 2(2rs + r + s) + 1 \quad \text{Basic Algebra}$$

$$\text{Let } k = (2rs + r + s) \quad \text{closure of } \mathbb{Z}$$

$$mn = 2k + 1 \quad \text{Def. of odd}$$

Q.E.D.