

Exam 2

p. 162 no. 49 "The difference of any two even integers is even."

Theorem: The difference of any two even integers is even

Proof: Let m and n be arbitrary even numbers such that $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$.

$$m = 2r \text{ for some } r \in \mathbb{Z}$$

Definition of even

$$n = 2s \text{ for some } s \in \mathbb{Z}$$

Definition of even

$$m - n = 2r - 2s$$

Substitution

$$m - n = 2(r - s)$$

Fundamental theorem of algebra

$$\text{Let } k = r - s \in \mathbb{Z}$$

Closure of integers in \mathbb{Z}

$$m - n = 2k$$

Even by definition of even