Exam #2 DAH 3/2/2011

THEOREM: #52

For all integers m, if $m > 2$ then $m^2 - 4$ is composite.

PROOF: Disproof by counterexample.

Find an integer $m > 2$ such that $m^2 - 4$ is prime.

Let $n = m^2 - 4$ where $n \in \mathbb{Z}$	(Closure of difference in $\mathbb{Z}$)
If n is prime, then $\forall$ positive integers r and s, if $n = r \cdot s$ then $r = 1$ or $s = 1$	(Definition of prime)
Let $m = 3$	(Counterexample)
$n = m^2 - 4$	
$= 3^2 - 4$	(Substitution)
$= 9 - 4$	(Basic algebra)
$n = 5$	
Let $r = 5$ and $s = 1$ such that $5 = 5 \cdot 1$	(Closure of multiplication)
Because 5 and 1 are the only tow distinct integers divisors of n and $r = 1$ or $s = 1$	(Definition for prime)
Therefore, 5 is prime and	(Conclusion)
$m^2 - 4$ is not composite for all integers $m > 2$	(Disproof)

G.E.D.