P. Staley
Southwestern College

M254 name:______________________________________
Exam 4.6-5.5 DAH version

Parts of this exam will be provided on the class website under the link Exam 3 DAH inserts. If any question below contains "RTWS" that means Refer To WebSite to fill in that part of the question. Be sure to use the RTWS material that corresponds to you SWC Student Id (sid).

Show your work on this exam. CIRCLE YOUR ANSWERS. Be neat. In all cases **justify your answers.**

1. If $(\mathbf{x})_B$ = __________________ (RTWS) is the coordinate vector of **x** relative to the basis B = ____ (RTWS) find the coordinate vector of **x** relative to the standard basis in $\mathbf{R}^3$.

2. Find the coordinate vector of the polynomial **p**(t) = _________________________ (RTWS) relative to the basis B = ____ (RTWS).

3. Find the unit vector in the direction opposite to that of _________________________ (RTWS).

4. Find the angle between the vectors ________________ and ________________________(RTWS).

5. Consider C[-1,1], the set of continuous functions on [-1,1], with the inner product

$< p,q > = \int_{-1}^{1} p(x)q(x)dx$. Determine whether the set {____________________________} (RTWS) is orthogonal, orthonormal, or neither. Justify your answer.

6. Find an orthonormal basis for the span of the set in #5 above.

7. For P_3, polynomials of degree three or less, and the inner product in #5 above, __________________________________ (RTWS) is an orthonormal basis.

Use this information to find the cubic polynomial least squares approximating function g(x) for the function f(x) = _________________ (RTWS) on the interval [-1, 1].
(Hint: Find the closest vector in this IP space.)

8. For the matrix

A = (RTWS)

Find the rank and a basis for the column space.

9. For the matrix

A = (RTWS)

Find a basis for, and dimension of, the solution space of AX = 0.

10. A Walsh basis for R^8 is given by

$W_1 = (1,1,1,1,1,1,1,1)$
$W_2 = (1,1,1,1,-1,-1,-1,-1)$
$W_3 = (1,1,-1,-1,-1,-1,1,1)$
$W_4 = (1,1,-1,-1,1,1,-1,-1)$
$W_5 = (1,-1,-1,1,1,-1,-1,1)$
$W_6 = (1,-1,-1,1,-1,1,1,-1)$
$W_7 = (1,-1,1,-1,-1,1,-1,1)$
$W_8 = (1,-1,1,-1,1,-1,1,-1)$

This basis is orthogonal with respect to dot product.

Find coordinate vector of

x = ______________________________(RTWS) relative to the Walsh basis.

11. A is an nxn matrix. List statements that are equivalent to A is invertible.

a. ____

b. ____

c. ____

d. ____

e. ____

f. ____

g. ____

h. ____

i. ____

j. ____

12. (Extra credit) In image processing, an 8 by 8 pixel grayscale image, i.e. an integer matrix in $M_{8,8}$, is often represented in the 2D Walsh basis given by $W_{i,j} = (W_i)^T (W_j)$. These matrices are orthogonal with respect to the Frobenius inner product (dot product as though they are vectors [google it if you need to]).

See if you can use this basis to represent the Happy Face:

°	°	■	■	■	■	°	°
°	■	°	°	°	°	■	°
■	°	■	°	°	■	°	■
■	°	°	°	°	°	°	■
■	°	■	°	°	■	°	■
■	°	°	■	■	°	°	■
°	■	°	°	°	°	■	°
°	°	■	■	■	■	°	°

use zero for °, and some positive value of your choice for ■. [Note for a black and white image use one for ■]