Random Walk on a One-Dimensional Path

Consider a particle on a line that at each time step moves either one unit to the left or one unit to the right. The choice of direction is independent of any prior steps. The probability of moving right at any time is r. The probability of moving left at any time is 1-r.

We would like to know for various values of r the probability, p_k, that a particle will eventually go k steps to the left of its starting position. What is the value of p_1 if r = 1? What is the value of p_1 if r = 0? What is p_2 if $p_1 = 1$? What is p_2 if $p_1 = 0$?

Exercise 1: Produce an Excel worksheet to simulate 100 particles (rows) taking 100 steps (columns) each. For each particle (row) determine its left most position (LMP) as the minimum of the position values in that row. With the initial position set to zero, estimate p_1 by counting all the LMP values less than zero (divide by the number of particles). Be sure that r_ is a labeled cell and that the recalculation mode is set to manual (tools-options-calculation tab-manual). You will most likely need the following Excel functions: IF(,,), RAND(), MIN(), and COUNTIF(,).

Exercise 2: Using the Excel sheet in exercise 1 estimate the value of p_1 for the following values of r: 1, .99, .9, .75, .5, .25, .01, 0. Use the recalculate key (F9) at least three times for each r value and average the results. This is equivalent to tracking 300 particles for 100 steps each.

Exercise 3: Modify the Excel sheet to estimate p_2 by counting LMP values less than –1. Repeat the runs in exercise 2 to estimate p_2 for each of the specified r values.

Exercise 4a: Consider the three mutually exclusive possibilities for a particle: (1) it nevers goes left of its starting position, (2) it goes left at most one position from its starting position, or (3) it goes left two or more positions from its starting position. Use this scenario to determine p_2 as a function of p_1.

Exercise 4b: The paths that eventually go one step left of the starting position are those that immediately go left (probability 1-r on the first step) or those that initially go right (probability r on the first step) and then eventually go left two steps from that position (probability p_2). Use this idea to derive another equation relating p_1, p_2, and r.

Exercise 5: Use exercises 4a and 4b to determine p_1 as a function of r. Notice that $0<=p_1<=1$ and that p_1 should go to zero as r goes to one. Use that result and 4a to generate a formula for p_2. Use an Excel sheet to compare the results of exercises 2 and 3 with the results predicted by your formula. Discuss the discrepencies. Continue the idea of exercise 4a to get a formula for p_k in terms of r.